INCORPORATING STABILITY, SURFACE LOCATION ERROR, TOOL WEAR, AND UNCERTAINTY IN THE MILLING SUPER DIAGRAM

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KEYWORDS
Machining, Chatter, Surface location error, Flank wear

ABSTRACT
An improved milling “super diagram” is presented that graphically depicts not only stability and surface location error limitations to milling productivity, but also incorporates the influence of tool wear and uncertainty. Combinations of axial depth of cut and spindle speed that offer stable cutting conditions with an acceptable, user-defined surface location error level are identified by a gray-scale color coding scheme. The effect of tool wear is included through the force model coefficients used for process dynamics prediction. User beliefs about data and model accuracy are applied to identify safety margins relative to the deterministic boundaries. A numerical case study and experimental wear results are included.

INTRODUCTION
Limitations to milling productivity include tool wear, positioning errors of the tool relative to the part, spindle error motions, fixturing concerns, programming challenges, and the process dynamics. Many research studies have been completed to address these issues. Taylor established an empirical basis for the relationships between cutting parameters and tool wear [Taylor]. Other issues are detailed in available texts, including [Trent and Wright; Tlusty; Altintas; Siocum; Dornfeld and Lee; Schmitz and Smith], for example.

In this work, the productivity limits imposed by both the process dynamics and tool wear are considered. For a particular milling system, the tool point frequency response function (FRF) and force model enable prediction of the stability limit and location of the machined surface due to forced vibrations (surface location error, or SLE) as a function of spindle speed and cut geometry. Frequency-domain solutions for the stability limit have been available for many years [e.g., Tlusty et al.; Altintas and Budak]. In these analyses, the data is presented graphically in the form of stability lobes, which separate stable combinations of spindle speed and axial depth from unstable pairs. More recently, a frequency-domain solution for SLE was also derived [Schmitz and Mann] to complement the well-known stability analyses and provide a complete picture of the dynamic limitations.
The effects of tool wear are incorporated here by applying wear status-dependent cutting force coefficients to the calculation of the stability limit and SLE. Additionally, model and input data uncertainties are included as a user-specified safety margin. The safety margin captures the user's belief regarding how close (in spindle speed and axial depth) he/she is willing to operate relative to the deterministic limits obtained using the frequency-domain models.

SUPER DIAGRAM DESCRIPTION

The initial super diagram combined stability and SLE information in a user-friendly graphical format [Zapata et al.] that relied on a gray-scale color coding scheme. The deterministic stability limit [Altintas and Budak] and SLE values [Schmitz and Mann] for a selected spindle speed-axial depth of cut domain were calculated based on the system dynamic response, radial depth of cut, and cutting force model. The threshold for acceptable SLE was user-defined.

To construct a diagram, the user selected the radial depth of cut, feed per tooth, SLE limit, and spindle speed-axial depth domain. This domain was then discretized into a grid of test points. A penalty was applied to each of the points as follows. Points that were stable and within SLE tolerance levels were not penalized; their value was set to zero. Points that were stable, but outside the SLE tolerance were penalized by one (value = -1). Unstable points were penalized by two (value = -2). The point values for the selected grid were then used to construct a contour plot that served as the super diagram. A gray-scale scheme was used to separate the different zones. The feasible zone (point values = 0) was white, the SLE-limited zones (-1) were gray, and the unstable zone (-2) was black.

In the new super diagram, the cutting force model provided in Eq. 1 is experimentally evaluated as a function of the wear status of the tool. This enables a diagram to be constructed for any level of tool wear depending on the volume of material removed, for example. In Eq. 1, \( F_r \) is the tangential force component, \( K_{t} \) is the tangential cutting force coefficient, \( b \) is the axial depth of cut, \( h \) is the instantaneous chip thickness (which depends on the feed per tooth), \( K_{n} \) is the tangential edge (plowing) coefficient, \( F_n \) is the normal force component, \( K_{n} \) is the normal cutting force coefficient, and \( K_{nc} \) is the normal edge coefficient [Altintas; Schmitz and Smith].

\[
F_r = K_{t}bh + K_{nc}b \quad F_n = K_{n}bh + K_{nc}b
\] (1)

It has been previously shown that the force model coefficients tend to increase as wear progresses, e.g., [Cui et al.]. By correlating the change in these coefficients with wear status, the diagram can be tailored to the behavior of a new tool or one at or near its end of life.

The limit between stable/unstable behavior, as well as between acceptable/unacceptable SLE values, was previously indicated using a binary format. In practice, however, there is uncertainty in the actual location of these boundaries and they should be represented by probability distributions [Abbasi et al.; Duncan et al.]. To incorporate uncertainty into the super diagram, a user-dependent safety margin is applied to modify the feasible (white) zone. The user selects how close in spindle speed and axial depth he/she is willing to operate relative to the predicted limits. Points within the feasible zone which violate this margin are penalized and a new "safe" feasible zone is identified. A second gray level is now incorporated. Dark gray indicates the stable points where the SLE limit is exceeded, while light gray represents the previously feasible points which violate the safety margin.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>Stiffness</td>
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<td>N/m</td>
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<tr>
<td>Damping ratio</td>
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<tr>
<td>Natural frequency</td>
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<td>Tool diameter</td>
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<tr>
<td>Helix angle</td>
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<td>deg</td>
</tr>
<tr>
<td>Number of teeth</td>
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<td></td>
</tr>
<tr>
<td>Tangential cutting coefficient, ( K_t )</td>
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<td>N/m²</td>
</tr>
<tr>
<td>Normal cutting coefficient, ( K_n )</td>
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<td>N/m²</td>
</tr>
<tr>
<td>Feed per tooth</td>
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<td>mm/rev</td>
</tr>
<tr>
<td>Radial depth of cut</td>
<td>4.7</td>
<td>mm</td>
</tr>
</tbody>
</table>

NUMERICAL CASE STUDY

To demonstrate the super diagram including tool wear and uncertainty, a numerical case study was completed. The parameters are given in Table 1. The original diagram for the selected tool wear value was created.
System is shown in Fig. 1, where the surface finish error limit is 50 μm.

Tool life is traditionally specified by the time required to reach a pre-selected wear level, often quantified using flank wear width (FWW) [Tayor, Trent, and Wright; Tlusty]. The FWW tends to increase with volume removed and is cutting speed (spindle speed) dependent. The assumed relationship between FWW and volume removed for the numerical example is shown in Fig. 2, where the tool life was defined as the time to reach 0.3 mm FWW.

The volume removed are provided in Figs. 2 and 3. The assumed linear relationship between these coefficients, spindle speed, \( \Omega \), and volume removed, \( V \), is provided in Eq. 2, where the intercepts, \( c_{01} \) and \( c_{02} \), are the new tool coefficients (Table 1), and \( c_{11} \) and \( c_{12} \) are the speed-dependent rates of increase with \( V \). These slopes were assumed to increase linearly with \( \Omega \) between 2000 rpm and 10000 rpm such that the coefficients doubled at 10000 rpm for \( V = 20 \) cm³ (where FWW = 0.3 mm; see Fig. 2) with no change at 2000 rpm for the same \( V \).

\[
K_1(\Omega, V) = c_{01} + c_{11}V \\
K_2(\Omega, V) = c_{02} + c_{12}V
\]  

As noted, the cutting forces tend to grow with FWW. The assumed relationships between the cutting coefficients, \( K_1 \) and \( K_2 \), identified in Eq. 1 and the volume removed at different spindle speeds are shown in Fig. 3 and 4.

Transactions of NAMRI/SME
Volume 38, 2010
As an example, for $\Omega = 3000$ rpm with $V = 20$ cm$^3$, $c_{uv} = c_{uv}^{3000-2000} = 1.25 \times 10^2$ Nm$^3$/cm$^3$ $V = 10000-2000$ by linear interpolation and $K_v = 2.0 \times 10^8 + 1.25 \times 10^7 V = 2.25 \times 10^9$ Nm$^2$. Given this relationship between cutting coefficients, $V$, and $\Omega$, the super diagram can then be modified to incorporate tool wear (the edge coefficients are not included in this example without loss of generality). As before, the $(\Omega, b)$ domain is represented by a grid of points and the stability and SLE is determined for each point. However, in this case, the volume to be removed must first be selected by the user. Then, the coefficients can be calculated for each spindle speed and, subsequently, the stability limit can be determined. Also, SLE is calculated at each axial depth grid point for the given spindle speed. The new diagram for $V = 20$ cm$^3$ is provided in Fig. 5. Because the cutting coefficients grow with $\Omega$, the stability limit decreases and the SLE infeasible zone grows while moving from left to right in the diagram.

Next, Fig. 5 can be modified to incorporate the user's uncertainty regarding the actual location of the deterministic boundaries. To carry out this task, the user defines safety limits, $\Delta\Omega$ and $\Delta b$, which give the distances from the boundaries that represent his/her 95% confidence level for feasible performance. For each feasible point in the $(\Omega, b)$ grid (white zone), the feasibilities (point values) of the surrounding eight points at distances $\Delta\Omega$ and $\Delta b$ from the test point are queried (see Fig. 6). If any of these points are infeasible, then the test point is penalized and also identified as infeasible. A new gray-scale is then implemented where the point values are feasible (0, white), safety margin (+, light gray), SLE limit (-, dark gray), and unstable (-, black). See Fig. 7 with $\Delta\Omega = 100$ rpm and $\Delta b = 0.5$ mm for the updated super diagram.

**FIGURE 6. SAFETY LIMIT IS IDENTIFIED BY TESTING THE FEASIBILITY OF THE EIGHT POINTS SURROUNDING $(\Omega, b)$: IN THIS CASE, THE TEST POINT IS PENALIZED (-) BECAUSE THE (BLACK) POINT ABOVE IT IS UNSTABLE.**

**FIGURE 7. SUPER DIAGRAM INCLUDING TOOL WEAR EFFECTS AND THE USER-DEFINED SAFETY MARGIN ($V = 20$ CM$^3$, $\Delta\Omega = 100$ RPM, and $\Delta b = 0.5$ MM).**

**EXPERIMENTAL TOOL WEAR RESULTS**

In this section, the experimental steps required to collect the tool wear data for the new super diagram are described for a 19 mm diameter inserted endmill (one square uncoated Kennametal 107888126 C9 JC carbide insert, zero rake and helix angles, 15 deg relief angle, 9.53 mm square x 3.18 mm) that was used to machine 1018 steel. An atomic force microscope (AFM) was used to measure the topography of the carbide. In this case, no digital readout (DRO) is available for the cutting tool, and hence the cutting tool may not move smoothly along the rake face.

**FIGURE 4. INSERT TOOLS REMOVAL MEASUREMENTS.**

The tool was removed from the insert and the carbide surface was then measured with an AFM.
Figure 8 shows an example 50 μm x 50 μm measurement (256 line scans, no digital filtering) of the rake face. It is seen that there is a small chamfer with a 167 deg angle at the cutting edge. The roughness average for the rake face was 310 nm.

In addition to monitoring the cutting force, the insert wear status was also measured at intervals. To avoid removing the insert/tool from the spindle, a handheld microscope (60x magnification) was used to record the rake and flank surfaces. The calibrated digital images were used to identify the FWW (no crater wear was observed). Example FWW results for 2500 rpm are provided in Fig. 9 (1σ error bars), where the wear increment is 12 cm³. Microscope images of the relief face for selected volumes of material removed are displayed in Fig. 10.

The cutting forces were monitored using a table-mounted force dynamometer (Kistler 9257B) and the four cutting force coefficients in Eq. 1 were identified intermittently while wearing the tool by performing a linear regression to the mean x (feed) and y direction forces over a range of feed per tooth values: {0.03, 0.04, 0.05, 0.06, and 0.07} mm/tooth [Altintas, Schmitz and Smith]. The feed per tooth while wearing the tool was kept constant at 0.06 mm/tooth. The radial and axial depths of cut were 4.7 mm (25% radial immersion) and 3 mm.

For each interval in Fig. 9, the four cutting force coefficients were also determined. These
results are given in Figs. 11 and 12. For the tool/material pair tested here, there was an approximately linear growth in $K_r$ and $K_v$, while the $K_{re}$ and $K_{te}$ values showed no clear trend.

![Graphs of $K_r$ and $K_v$ versus volume removed](image)

**FIGURE 12. VARIATION IN $K_r$ AND $K_v$ WITH VOLUME REMOVED ($\Omega = 2500$ RPM).**

![Graphs of $K_{re}$ and $K_{te}$ versus volume removed](image)

**FIGURE 13. VARIATION IN $K_r$ AND $K_v$ WITH VOLUME REMOVED FOR VARIOUS SPINDLE SPEEDS.**

To evaluate the change in coefficient behavior with spindle speed, additional tests were completed at (3750, 5000, 6250, and 7500) rpm. The results are displayed in Fig. 13, as well as the linear least squares fits. As expected, the rates of $K_r$ and $K_v$ growth (i.e., the slopes) increase with spindle speed. Interestingly, when plotted versus the corresponding FWW (measured with the handheld digital microscope), the five different spindle speed results collapse onto a single line; see Fig. 14. This suggests that if the FWW were monitored, it could provide an in-process approach to updating the force model coefficients based on the tool wear status. Finally, while the results are not shown, the $K_{re}$ and $K_{te}$ values again did not exhibit any significant trend at the additional spindle speeds.

![Graphs of $K_r$ and $K_v$ versus FWW](image)

**FIGURE 14. VARIATION IN $K_r$ AND $K_v$ WITH FWW AT VARIOUS SPINDLE SPEEDS.**

The slopes of the individual ($K_r$ and $K_v$ versus volume removed) lines in Fig. 13 are plotted against spindle speed in Fig. 15. It is seen that these slopes grow linearly with increasing speed, as was assumed in the numerical example. Therefore, the Eq. 2 model, which linearly relates the $K_r$ and $K_v$ values to $\Omega$ and $V$, can be applied for this tool-material pair. The slope with lines in Fig. 15 are $7.1 \times 10^{-4}$ (N/m²/cm³)/rpm for the $K_r$ data and $9.1 \times 10^{-4}$ (N/m²/cm³)/rpm for the $K_v$ data. Using the Eq. 2 notation, the $c_{01}$ and $c_{1r}$ values can then be calculated by multiplying these rates of change by the spindle speed of interest. Also, the intercepts from Eq. 2, $c_{01}$ and $c_{1r}$, are determined from the mean $V = 0$ values from Fig. 13. Given this information, the Eq. 2 model can be populated as shown in Eq. 3. Because a significant trend in $K_{re}$ and $K_{te}$ was not observed, these values can be set equal to the mean values from Fig. 12, i.e., $K_{re} = 4.6 \times 10^{3}$ N/m and $K_{te} = 3.9 \times 10^{3}$ N/m.

$$K_r(\Omega, V) = 2.2 \times 10^{3} + (7.1 \times 10^{-4}) \Omega$$

$$K_v(\Omega, V) = 1.2 \times 10^{3} + (9.1 \times 10^{-4}) \Omega$$
FIGURE 15. VARIATION IN SLOPE OF K₁ AND K₂, VERSUS VOLUME REMOVED LINES (FROM FIG. 13) WITH SPINDLE SPEED.

Given this relationship, the super diagram that incorporates tool wear can then be developed by applying the user-selected volume and calculating K₁ and K₂ for each spindle speed in the \((Ω, b)\) domain as shown in the numerical example.

![Graph showing variation in K₁ and K₂ with volume removed for various feed per tooth values.](image)

FIGURE 16. VARIATION IN K₁ AND K₂ WITH VOLUME REMOVED FOR VARIOUS FEED PER TOOTH VALUES.

All previous testing was carried out using a feed per tooth value of 0.06 mm/tooth. This enables a single super diagram to be developed. If the user-selected feed per tooth value was changed, however, a new diagram is required. Therefore, experiments were completed at feed per tooth values of 0.03, 0.045, 0.075, and 0.09 mm/tooth. The spindle speed was 5000 rpm and the other parameters remained the same. Figure 16 shows the corresponding variation in K₁ and K₂, It is seen that the wear rate is higher and the volume of material that can be removed is lower for the smaller feed per tooth values. The wear rate trend suggests that strain hardening may be in effect. The thinner chips with increased hardness can cause accelerated wear. The reduced amount of material that can be removed could also be attributed to the increased number of passes through the material required to remove the same volume (in addition to strain hardening).

Finally, the potential for variation in wear rate behavior with axial depth of cut was evaluated. The axial depths were \(3, 4.5,\) and \(6\) mm, the spindle speed was 5000 rpm, the feed per tooth was 0.06 mm/tooth, and the radial depth remained at 4.7 mm. It is observed in Fig. 17 that the three test sets collapse onto a single line when plotted versus the normalized volume, \(V_n = V/b\). This normalization was necessary because the independent variable, \(V_n\), is a function of the dependent variable, \(b\). The agreement between \(b\) values demonstrates that testing at a single axial depth is sufficient. The divergence at the highest \(V_n\) value for \(b = 6\) is due to excessive FWW for that test (> 0.7 mm). Similar results are obtained when the radial depth of cut is varied.

![Graph showing variation in K₁ and K₂ with normalized volume removed for various axial depths of cut.](image)
CONCLUSIONS

The new comprehensive milling super diagram was described that provides information at the process planning stage for stability, surface location error, tool wear, and uncertainty in a user-friendly graphical format. The grayscale color scheme identifies: 1) stable combinations of axial depth of cut and spindle speed that offer both stable cutting conditions and an acceptable, user-defined surface location error level within a user-selected safety margin (white); 2) stable cutting conditions that meet the deterministic SLE limit but are not within the safety margin (light gray); 3) stable cutting conditions that do not meet the surface location error limit (dark gray); and 4) unstable cutting conditions (black).

A numerical case study was presented to describe the diagram development and tests were completed to establish the variation in cutting force coefficients with tool wear as a function of spindle speed and volume removed for a zero rake and helix angles, 15 deg relief angle, square, uncoated carbide insert used to machine 1018 steel. The single insert was mounted in a 19 mm diameter steel tool body. For this tool-material pair, it was observed that the cutting coefficients, which relate the tangential and normal force components to chip area, increased linearly with volume removed and the corresponding slope increased linearly with spindle speed. However, the edge (plowing) coefficients, that relate the forces to chip width only, showed no appreciable trend with tool wear.

ACKNOWLEDGMENTS

The authors gratefully acknowledge financial support for this work from the National Science Foundation (CMMI-0926667) and Kennametal, Inc. Any opinions, findings, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of these agencies.

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