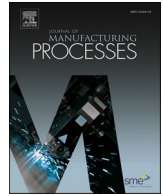




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## Physics-informed KNN milling stability model with process damping effects

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## ABSTRACT

This paper describes a k-nearest neighbors, or KNN, model for milling stability including process damping effects. A physics-based, frequency domain milling stability solution is used to generate the training data, but does not incorporate process damping effects. The data set is then updated using limited tests to capture the process damping behavior. A “stair step” approach is used to select the test points, where a first spindle speed-axial depth combination is selected based on the physics-based stability map, subsequent tests are defined using the previous test result, and data points are updated by knowledge of process damping behavior and the test results. The KNN modeling approach demonstrates the ability to predict both stable and unstable results, including process damping behavior.

## 1. Introduction

Machining and machine tool technology continue to advance in order to meet industry and government manufacturing requirements. Despite these continuing increases in productivity, challenges remain. For example, while the digital transformation of part design and path planning for computer numerically controlled (CNC) machining is ubiquitous, computer aided manufacturing (CAM) software generally treats machining as a geometric activity. Provided the cylindrical tool follows the required path through the stock model, which may be obtained from the structured light scan of an additive manufacturing preform [1–3], it is assumed that the machining process is acceptable and the desired geometry is obtained. This approach does not consider the inherent constraints imposed by the process dynamics.

It is well known that some spindle speed-axial depth of cut combinations exhibit self-excited vibration (or chatter, which produces large forces, large vibrations, and poor surface finish), while others do not [4–6]. Additionally, even if stable behavior is obtained (i.e., forced vibration only), surface location error may affect the geometric accuracy of the machined part, again depending on the selected spindle speed-axial depth combination [4,7–10]. Machining dynamics models are therefore required to select spindle speed-axial depth combinations that avoid chatter, while meeting design tolerances.

Frequency domain solutions for milling stability maps, which separate stable spindle speed-axial depth combinations from those that produce chatter, have been presented and validated in the literature [11,12]. These stability maps demonstrate large stable zones at high

spindle speeds. When tool wear constrains the maximum cutting speed, however, lower spindle speeds must be selected. While the frequency domain stability solutions predict small stable depths of cut at low spindle speeds, it is known that process damping can increase the stability limit in this low spindle speed zone. Many machining researchers have studied the process damping effect. Early studies were completed by Wallace and Andrew [13], Sisson and Kegg [14], Peters et al. [15], and Tlustý [16]. Although initial efforts have been made to incorporate process damping in the frequency domain stability solution [17–20], a comprehensive analytical solution is not widely available.

Recent advances in machine learning algorithms and increases in computing power and data storage have accelerated the application of machine learning to machining modeling, including process stability. One motivation for the use of data-driven learning algorithms is the presence of uncertainty in physics-based stability predictions due to uncertainties in the model inputs (tool tip frequency response functions, cutting force coefficients, and process damping coefficients). Multiple authors have studied the application of machine learning to machining stability. Cherukuri et al. applied an artificial neural network (ANN) to model stability in turning [21]. Denkena et al. implemented support vector machines and ANNs [22]. Bergmann and Reimer applied Regularized Kernel Interpolation for a learning stability map [23]. Bayesian machine learning approaches have also been evaluated [24–28]. For example, Schmitz et al. described a milling stability identification approach that simultaneously considered physics-based models for the tool tip frequency response functions and stability predictions; the binary result from a milling test; chatter frequency when an unstable result

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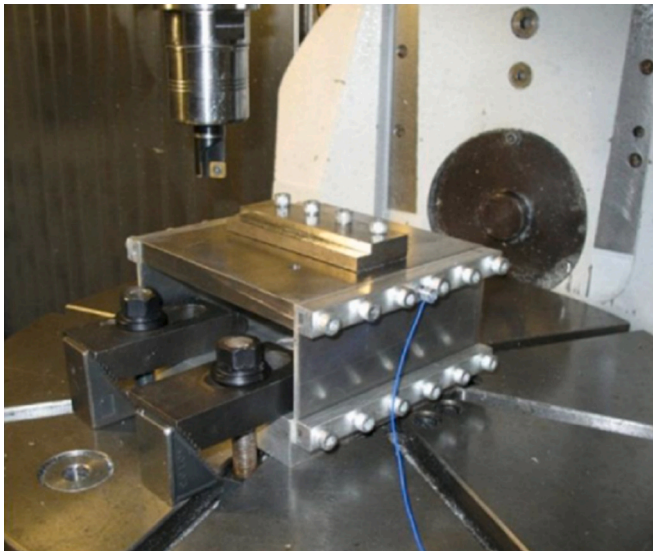


Fig. 1. Setup for milling stability tests. An accelerometer was used to measure the flexure motion during milling tests.

Table 1  
Modal parameters for flexure.

Direction	Natural frequency (Hz)	Modal stiffness (N/m)	Viscous damping ratio (-)
x	228	$2.77 \times 10^6$	0.063
y	1482	$1.74 \times 10^8$	0.037

was obtained; and user risk tolerance [28]. The algorithm applied Bayesian machine learning with adaptive, parallelized Markov Chain Monte Carlo sampling to update the probability of stability after each milling test.

Process damping has also been considered in machine learning models. Karandikar et al. used a random walk method for Bayesian inference to identify the process damping coefficient in milling [29]. An analytical process damping algorithm was used to model the prior distribution of the stability boundary locations and was updated using experiments. Karandikar et al. also described a value of information-based experimental design method that used Bayesian inference to update the process damping model [30]. Corson et al. detailed a physics-informed Bayesian machine learning approach, where they applied three physics-based models to establish the initial probability of milling stability. These models included: receptance coupling substructure analysis (RCSA) prediction for the tool tip frequency response functions; finite element software prediction of the mechanistic force model coefficients; and a spindle speed-dependent power law model for process damping [31].

This paper builds on the prior efforts in milling stability modeling, process damping, and machine learning by demonstrating a k-nearest neighbors, or KNN, algorithm to model milling stability with process damping effects. A KNN is a non-parametric, supervised learning classifier that uses proximity to predict the behavior at selected data point based on the surrounding points [32]. The assumption is that similar points are found near one another. For the milling stability classification problem studied here, a class label (stable or unstable) is assigned on the basis of a majority vote. In other words, the label that is most frequently represented around a given spindle speed-axial depth combination is used to predict the behavior at that point. The advantage of the KNN approach is its ease of implementation and low computation expense.

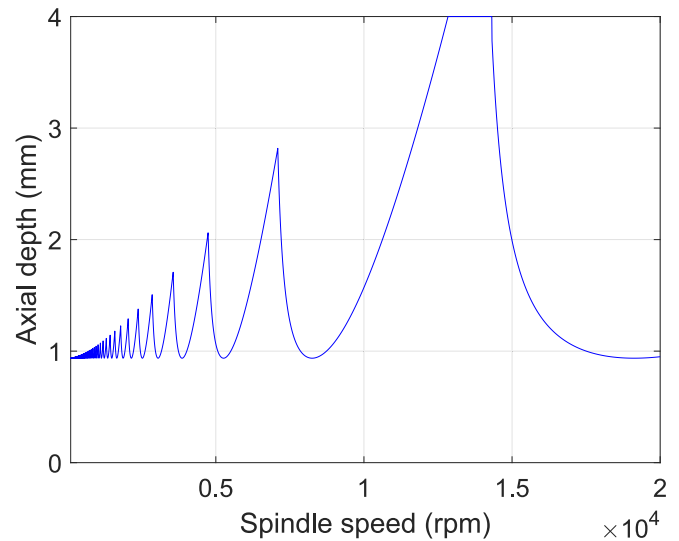


Fig. 2. Physics-based stability map for measured system dynamics and cutting force model (no process damping). Spindle speed-axial depth combinations above the stability boundary (blue line) are predicted to be unstable. Combinations below the boundary are predicted to be stable. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

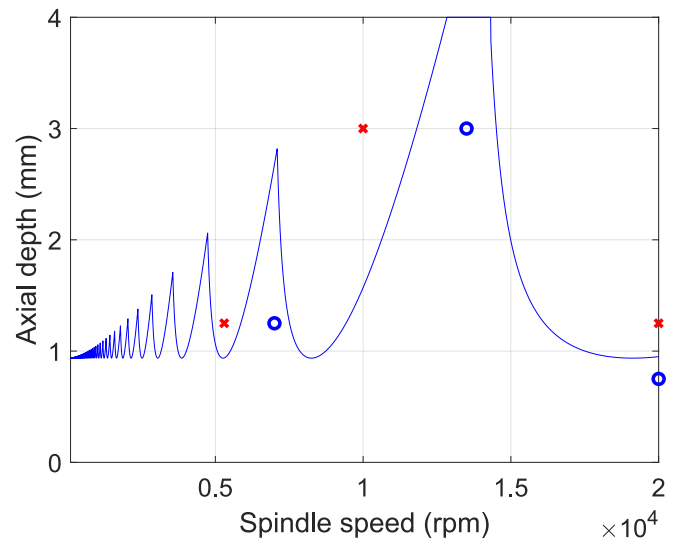


Fig. 3. Stability map with initial cutting test results. Blue circles represent stable tests. Red crosses represent unstable (chatter) tests. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

## 2. Stability evaluation

As demonstrated in [17], a flexure-based setup was used to collect milling stability data. A single degree-of-freedom, parallelogram leaf-type flexure provided a flexible base for an AISI 1018 steel workpiece as shown in Fig. 1. The flexure's dynamic stiffness was much lower than the tool tip dynamic response, so the average force direction frequency domain stability analysis [4] was completed using only the flexure's dynamic properties. The flexure's modal parameters are listed in Table 1. The x and y directions correspond to the flexible and stiff directions of the flexure, where x is the feed direction for the 50 % radial immersion up milling tests. The feed per tooth was 0.05 mm for all tests. The tool was a single-tooth indexable square end mill with a 18.54 mm

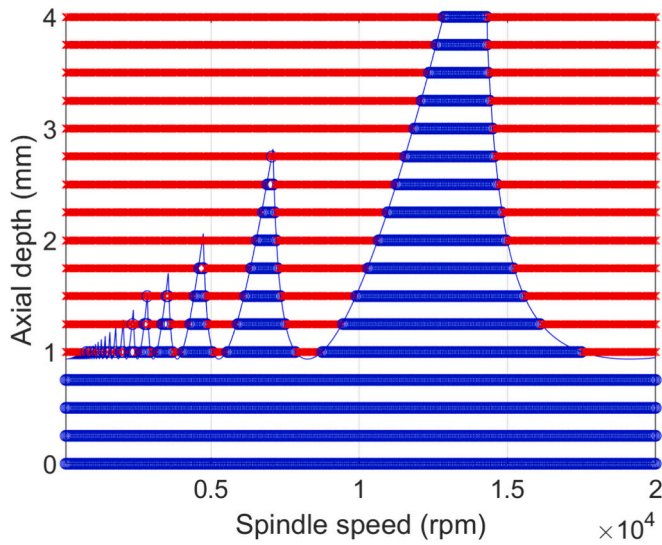


Fig. 4. Data set for KNN training based on physics-based stability map displayed in Fig. 2.

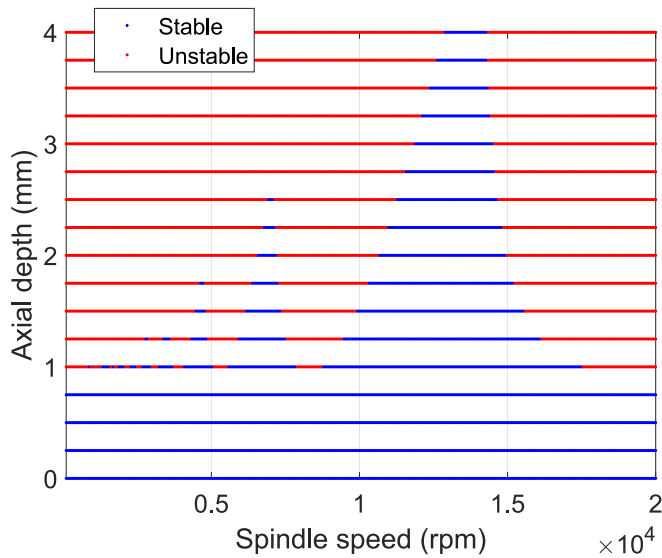


Fig. 5. KNN classifier for milling stability with no process damping.

diameter, 15 deg. relief angle, 0 deg. rake angle, and no edge preparation (Kennametal model KICR-0.73-SD3-033.3C).

The cutting force coefficients were identified under stable cutting conditions using a cutting force dynamometer (Kistler model 9257B). The specific cutting force,  $K_s$ , and cutting force direction,  $\beta$ , were determined to be 2359.1 N/mm<sup>2</sup> and 63.5 deg. using a linear regression to the mean cutting force over a range of feed per tooth values [4].

The stability map for the system dynamics described in Table 1 and the measured cutting force parameters is displayed in Fig. 2. The frequency domain model applied here did not include process damping effects. It is observed that the stability limit converges to approximately 1 mm at low spindle speeds.

To partially validate Fig. 2 stability map, milling tests were completed at selected stable and unstable spindle speed-axial depth combinations. In each instance, the predicted and measured stability

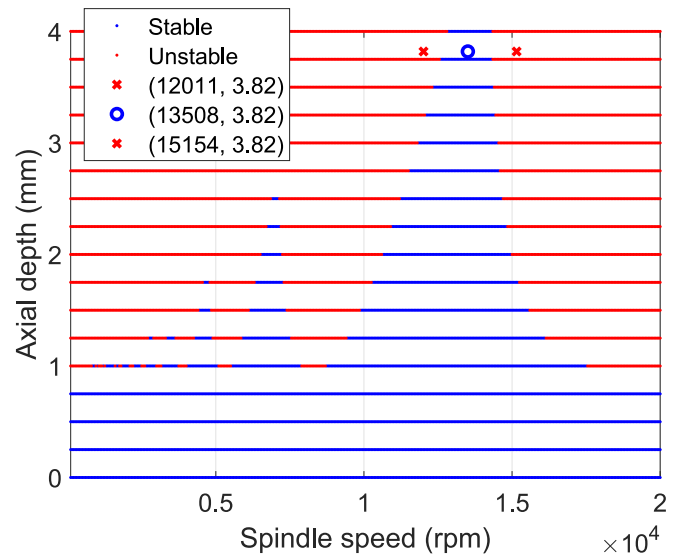


Fig. 6. KNN classifier with three predictions. The blue circle represents a stable prediction. The red crosses represent unstable predictions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

behavior agreed. The results are displayed in Fig. 3, where the blue circles represent stable test results and the red crosses represent unstable test results. The stable or unstable label was selected based on the frequency content of the vibration signal obtained from the accelerometer attached to the flexure. A cut was considered stable when the primary frequency content was observed at the tooth passing frequency and its multiples. A cut was considered unstable when another frequency (chatter frequency) was identified.

### 3. KNN classifiers

Given the initial stability boundary validation, a KNN classifier was developed without considering process damping. The first step was to specify a training data set using the analytical stability boundary from Fig. 2. This data set was prepared using a grid of points with increments of 50 rpm in spindle speed and 250  $\mu$ m in axial depth. Points below the stability boundary were labeled ‘stable’ and those above the boundary were labeled ‘unstable’. See Fig. 4, where the blue circles represent stable spindle speed-axial depth combinations and red crosses represent unstable combinations.

The *fitknn* function in MATLAB was used to generate the KNN classifier (stability model) with five neighbors (points), where the Euclidean distance was applied to identify the nearest neighbors. In order to approximately match scaling in the vertical (axial depth) and horizontal (spindle speed) axes, the axial depth was converted to units of  $\mu$ m. The KNN classifier is displayed in Fig. 5, where the same grid locations from Fig. 4 are used to show the classifier values (blue dot = stable, red dot = unstable).

To demonstrate the use of the classifier at arbitrary points, the behavior was predicted for three spindle speed-axial depth combinations not contained in the training data set. These were (12,011 rpm, 3.82  $\times$  10<sup>3</sup>  $\mu$ m), (13,508 rpm, 3.82  $\times$  10<sup>3</sup>  $\mu$ m), and (15,154 rpm, 3.82  $\times$  10<sup>3</sup>  $\mu$ m); see Fig. 6. The corresponding KNN classifier labels were ‘unstable’, ‘stable’, and ‘unstable’, as expected based on the physics-based stability map in Fig. 2.

The process damping behavior was next established using a “stair

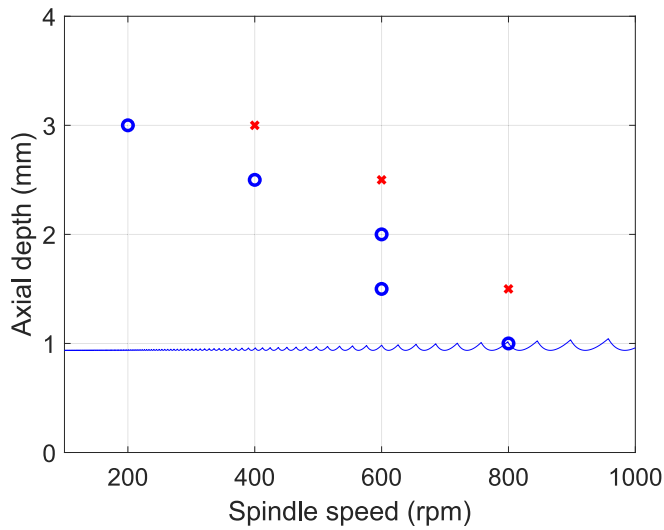


Fig. 7. Process damping test results. Blue circles represent stable tests. Red crosses represent unstable (chatter) tests. The original analytical stability boundary is included (solid blue line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2  
Process damping tests and results.

Test	Spindle speed (rpm)	Axial depth (mm)	Result
1	800	1	Stable
2	800	1.5	Unstable
3	600	1.5	Stable
4	600	2	Stable
5	600	2.5	Unstable
6	400	2.5	Stable
7	400	3	Unstable
8	200	3	Stable

step” approach with a minimum number of cutting tests. Based on the low flexure natural frequency (228 Hz), a relatively low spindle speed of 800 rpm was selected for the first test. A conservative axial depth of 1 mm was chosen given the predicted stability map (with no process damping) shown in Fig. 2. Based on the result of a test, a predefined progression was followed:

- If the cut was stable, the spindle speed was maintained and the axial depth was increased by 0.5 mm (50 % of the original axial depth). This was repeated until an unstable result was achieved.
- When an unstable cut was obtained, the spindle speed was reduced by 200 rpm (25 % of the original spindle speed) and the previous unstable cutting depth was selected.
- The sequence was repeated.

The test results are displayed in Fig. 7. The sequence of tests and results are listed in Table 2. Testing was terminated at a spindle speed of 200 rpm and axial depth of 3 mm.

Next, knowledge about process damping was applied using Fig. 7 test points to update the grid of points in Fig. 4. For a stable cut at a selected spindle speed-axial depth, all points with an equal or smaller axial depth and equal or lower spindle speed were labeled as stable. This provided many updating points from a single test and contributed to the “stair step” updating; see Fig. 8 for the original and re-labeled points.

To further examine the “stair step” updating, Fig. 9 shows the stable re-labeling decisions based on the test results and process damping knowledge. The blue box of points was updated using the stable result at 800 rpm and 1 mm. All points in the box were labeled as ‘stable’ given the stable test result (equal or lower spindle speeds and axial depths). The yellow box was updated using the stable result at 600 rpm and 2

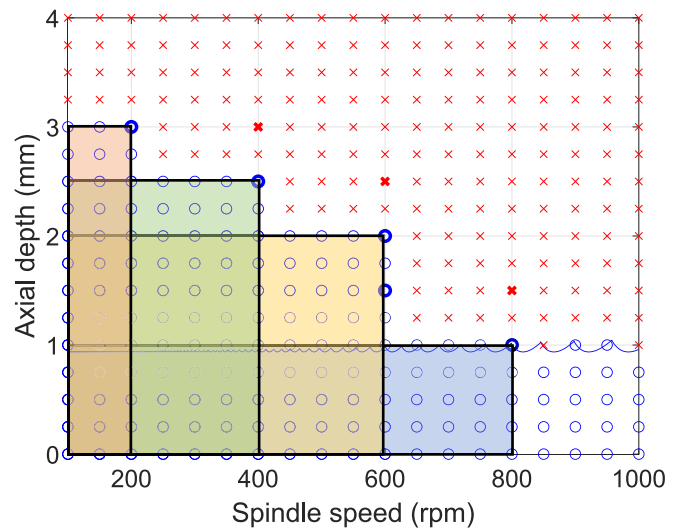


Fig. 9. “Stair step” updating description.

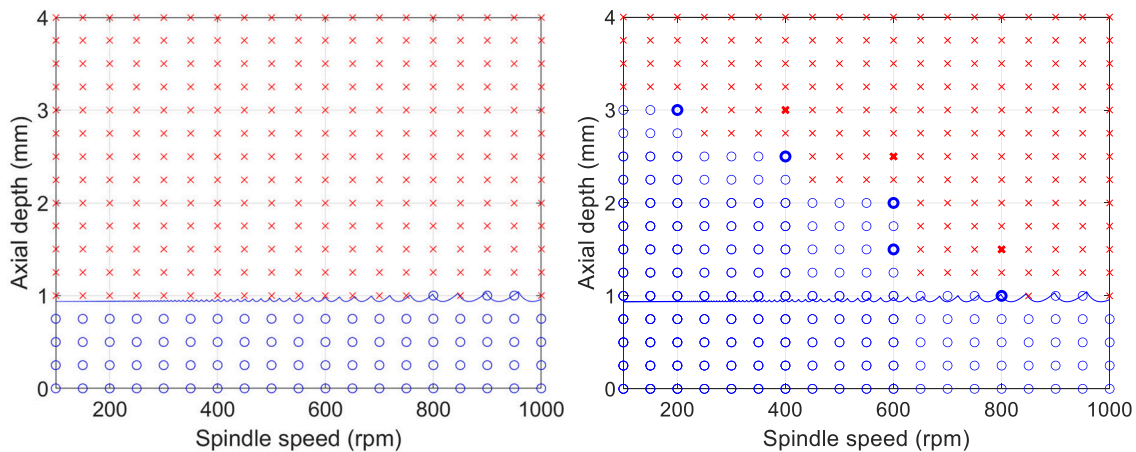


Fig. 8. (Left) original data points based on physics-based stability boundary. (Right) new data points with process damping effects. The test points from Fig. 7 are identified using the symbols with thicker lines.

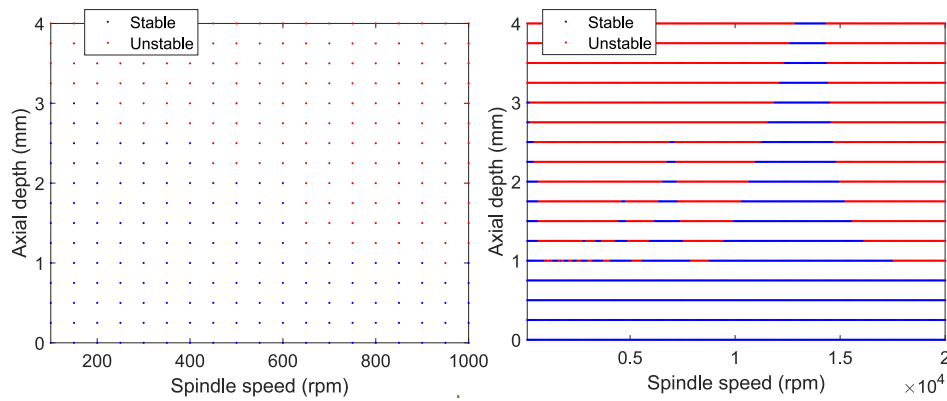


Fig. 10. KNN classifier for milling stability with process damping. (Left) low spindle speed range. (Right) full spindle speed range.

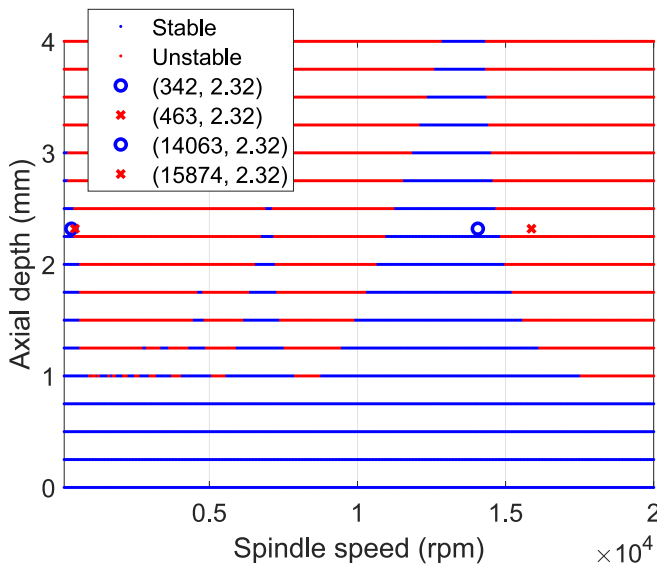


Fig. 11. KNN classifier including process damping with four predictions.

mm. The green box was updated using the stable result at 400 rpm and 2.5 mm. The orange box was updated using the stable result at 200 rpm and 3 mm.

The *fitknn* function in MATLAB was again used to generate the KNN classifier with five neighbors, but this time including process damping. The new KNN classifier is displayed in Fig. 10, where the same grid locations from Fig. 9 (left panel) and Fig. 6 (right panel) are used to show the classifier values (blue dot = stable, red dot = unstable).

To demonstrate the use of the classifier at arbitrary points, the behavior was predicted at four spindle speed-axial depth combinations not contained in the training data set. These were (342 rpm,  $2.32 \times 10^3 \mu\text{m}$ ), (463 rpm,  $2.32 \times 10^3 \mu\text{m}$ ), (14,063 rpm,  $2.32 \times 10^3 \mu\text{m}$ ), and (15,874 rpm,  $2.32 \times 10^3 \mu\text{m}$ ); see Fig. 11. The corresponding KNN classifier labels were ‘stable’, ‘unstable’, ‘stable’, and ‘unstable’, as expected based on the test results in Figs. 3 and 7.

#### 4. Conclusions

In this study, a k-nearest neighbors, or KNN, algorithm was used to model milling stability with process damping effects. The intent was to combine existing physics-based, frequency domain milling stability solutions, which do not include process damping, with a data-driven learning algorithm to incorporate process damping at low spindle speeds. To proceed, a physics-based, frequency domain milling stability solution was used to generate a training data set for the KNN model. This

data set was then updated using limited tests to capture the process damping behavior.

A “stair step” approach was used to select the test points. In this approach, a first spindle speed-axial depth combination was selected based on the original, physics-based stability map (without process damping). Follow-on tests were based on the previous test result using the following rule set: 1) If the cut was stable, the spindle speed was maintained and the axial depth was increased by a conservative amount (0.5 mm). This was repeated until an unstable result was achieved. 2) When an unstable cut was obtained, the spindle speed was reduced by a preselected increment (200 rpm, or 25 % of the original spindle speed) and the previous unstable cutting depth was selected. 3) The sequence was repeated.

The KNN modeling approach demonstrated the ability to predict both stable and unstable behavior, while incorporating process damping effects. Further, the number of required test points was small (only eight tests were necessary) due to the point updating strategy which relied on knowledge of the process damping behavior. Specifically, for a stable cut at a selected spindle speed-axial depth, all points with an equal or smaller axial depth and equal or lower spindle speed were also labeled as stable. This contributed to the “stair step” updating and limited the number of test points. While prior efforts have also applied machine learning to milling stability prediction, as well as the effect of process damping, the KNN approach implemented here enables straightforward implementation at low computational expense.

#### CRediT authorship contribution statement

**Tony Schmitz:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Validation, Visualization, Writing – original draft, Writing – review & editing.

#### Declaration of competing interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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