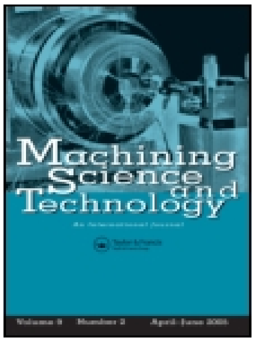

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TOOL LIFE PREDICTION USING RANDOM WALK BAYESIAN UPDATING

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□ *According to the Taylor tool life equation, tool life reduces with increasing cutting speed. The influence of additional factors can also be incorporated. However, tool wear is generally considered a stochastic process with uncertainty in the model constants. In this work, Bayesian inference is applied to predict tool life for milling/turning operations using the random walk/surface methods. For milling, Bayesian inference using a random walk approach is applied to the well-known Taylor tool life model. Tool wear tests are performed using an uncoated carbide tool and* AISI 1018 *steel work material. Test results are used to update the probability distribution of tool life. The updated beliefs are then applied to predict tool life using a probability distribution. For turning, both cutting speed and feed are considered. Bayesian updating is performed using the random surface technique. Turning tests are completed using a coated carbide tool and forged* AISI 4137 *chrome alloy steel. The test results are applied to update the probability distribution of tool life and the updated beliefs are used to predict tool life. While this work uses the Taylor model, by following the procedures described here, the technique can be applied to other tool life models as well.*

Keywords bayesian updating, random walk, taylor tool life, tool wear, uncertainty

INTRODUCTION

Tool wear can impose a significant limitation on machining productivity, particularly for hard-to-machine materials. Taylor (1906) first defined an empirical relationship between tool life and cutting speed using the power law:

$$VT^n = C \tag{1}$$

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where V is the cutting speed in m/min, T is the tool life in min, and n and C are constants which depend on the tool-workpiece combination. The constant C is defined as the cutting speed required to obtain a tool life of 1 minute. Tool life is typically defined as the time required to reach a predefined level of wear for a selected feature, such as flank wear width (FWW), crater depth, or notch depth depending on the nature of the tool wear. The Taylor tool life equation can be modified to include other effects, such as feed rate (Tlusty, 2000):

$$V^p f_r^q T = C \tag{2}$$

where f_r is the feed per revolution in mm/rev for turning and C, p, and q are constants which depend on the tool-workpiece combination and other cutting conditions. The Taylor tool life model is deterministic in nature, but uncertainty exists due to: 1) factors that are unknown or not included in the model; and 2) tool-to-tool performance variation. For these reasons, tool wear is often considered to be a stochastic and complex process and, therefore, difficult to predict.

Bayesian inference, which forms a normative and rational method for belief updating is applied in this work. Let the prior distribution about an uncertain event, A, at a state of information, &, be $\{A|\&\}$, the likelihood of obtaining an experimental result B given that event A occurred be $\{B|\mathrm{A},\&\}$, and the probability of receiving experimental result B (without knowing A has occurred) be $\{B|\&\}$. Bayes' rule is used to determine the posterior belief about event A after observing the experiment results, $\{A|B,\&\}$ as shown in Equation (3). Using Bayes' rule, information gained through experimentation can be combined with the prior prediction about the event to obtain a posterior distribution.

$$\{A|B,\&\} = \frac{\{A|\&\}\{B|A,\&\}}{\{B|\&\}} \tag{3}$$

As seen in Equation (1), the Taylor tool life model assigns a deterministic value to tool life for a selected cutting speed based on the empirical values n and C. In contrast, Bayesian inference assigns a probability distribution to the tool life value at a particular cutting speed. From a Bayesian standpoint, an uncertain variable is treated as random and is characterized by a probability distribution.

The prior, or initial belief of the user, can be based on theoretical considerations, expert opinions, past experience, or data reported in the literature; the prior should be as chosen to be as informative as possible. The prior is represented as a probability distribution and, using Bayes' theorem, the probability distribution is updated when new information becomes available (from experiments, for example). As a result, Bayesian inference enables a model to incorporate uncertainty in terms of a probability distribution and beliefs about this distribution to be updated based on experimental results (Gelman et al., 2009; Grimmett and Stirizaker, 2004).

The objective of the article is to demonstrate the random walk/random surface method for Bayesian updating and its application to tool life prediction. The Taylor tool life model is used in this study, despite its potential limitations, because it is well-known and generally understood in the manufacturing community. Without loss of generality, Bayesian updating using the random walk/random surface approach can be applied to other available models. The remainder of the article is organized as follows. The authors describe Bayesian updating of tool life in milling using the random walk method for the Taylor tool life model given by Equation (1). The experimental setup and tool life predictions are also provided. Next, we describe the random surface method of Bayesian updating for tool life in turning using the Taylor-type tool life model defined by Equation (2). Then, we cover the influence of the prior and likelihood on tool life predictions and draw conclusions from our research.

BAYESIAN INFERENCE OF THE TAYLOR TOOL LIFE MODEL FOR MILLING USING THE RANDOM WALK METHOD

Bayesian inference provides a rigorous mathematical framework of belief updating about an unknown variable when new information becomes available. In the Taylor tool life model [Equation (1)], there is uncertainty in the values of the exponent, n, and the constant, C. Subsequently, there is uncertainty in the tool life, T. The Taylor tool life curve can be predicted by generating N sample tool life curves, or sample paths, each representing the true tool life curve with an equal probability of $1/N$. The sample paths generated in this way may be used as the prior for Bayesian inference. The prior can then be updated by applying Bayes' rule to experimental test results. For each sample path, Bayes' rule can be written as the following product.

$$p(path = true\ tool\ life\ curve | test\ results) = \frac{p(test\ result | path = true\ tool\ life\ curve)}{p(test\ result)} \cdot p(path = true\ tool\ life\ curve)$$

Here, $p(path = true\ tool\ life\ curve)$ is the prior probability that a given path is the true tool life curve. As noted, the probability is assumed to be $1/N$ before any testing is completed since each path is considered equally likely to be the true tool life curve. Also, $p(test\ result \mid path = true\ tool\ life$

curve) is referred to as the likelihood, *p*(*test result*) is a normalization constant, and *p*(*path* = *true tool life curve* | *test result*) is the posterior probability of the sample tool life curve given a test result.

In this study, the prior sample paths were generated using random samples from an $\{n,C\}$ joint probability density function (pdf). The initial (prior) n and C distributions were selected based on a literature review. In general, the decision maker should try to use all available information to generate the sample paths. Bayes' rule was then used together with experimental results to update the probability that each sample path was the true tool life curve.

According to Bayes' rule, the posterior distribution is proportional to the (normalized) product of the prior and the likelihood. For multiple experimental results, the posterior after the first update becomes the prior for the second update and so on, where the posterior probabilities of each sample path must be normalized so that the sum of the probabilities for all paths is one. In a milling operation, other factors, such as feed rate and axial/radial depths of cut, may also affect tool life in addition to the cutting speed. However, since cutting speed is typically the strongest factor, Bayesian updating was performed using Equation (1).

Establishing the Prior

Tool wear experiments were performed using an uncoated carbide (inserted) tool to mill *AISI 1018* steel. As noted, a literature review was completed to determine the prior distributions of the Taylor tool life model values, n and C. Stephenson and Agapiou (2006) reported the value of n to be in the range of 0.2 to 0.25 for uncoated carbide tools and C to be around 100 for rough finishing of low carbon steels. Kronenberg (1966) reported values of n and C to be in the range of 0.3 to 0.5 and 160 to 200, respectively, for machining steel with a carbide tool. Creese (1999) reported typical n and C values for machining medium carbon steel with a carbide tool to be 0.32 and 240, respectively. Cui et al. (2009) performed wear experiments using a carbide insert and *1018* steel workpiece. Values of n and C were reported to be 0.3 and 341, respectively. In a separate study conducted by the authors, the mean n and C values for the given tool-work piece combination were found to be 0.33 and 600 (Karandikar et al., 2011).

Based on these values, the priors for n and C were selected to be uniform distributions with minimum values of 0.3 and 400, respectively, and maximum values of 0.35 and 700, respectively. A uniform distribution implies that it is equally likely for the true n and C value to be anywhere in the selected range. This is expressed using:

$$n = U(0.3, 0.35) \text{ and } C = U(400, 700)$$

TABLE 1 Prior probabilities and tool life for sample $\{n,C\}$ pairs

Sample	$\{n,C\}$	Tool life (min) 2500 rpm 149.6 m/min	5000 rpm 299.2 m/min	7500 rpm 448.9 m/min	Prior
1	{0.30, 500}	55.8	5.5	1.4	0.10
2	{0.30, 525}	65.7	6.5	1.7	0.10
3	(0.30, 550}	76.7	7.6	2.0	0.10
4	{0.30, 575}	88.9	8.8	2.3	0.10
5	(0.30, 600}	102.5	10.2	2.6	0.10
6	{0.35, 500}	31.4	4.3	1.4	0.10
7	{0.35, 525}	36.1	5.0	1.6	0.10
8	(0.35, 550}	41.2	5.7	1.8	0.10
9	{0.35, 575}	46.8	6.5	2.0	0.10
10	(0.35, 600}	52.9	7.3	2.3	0.10

where U denotes a uniform distribution and the values in the parentheses identify the minimum and maximum values, respectively.

The relatively large prior distributions of n and C were chosen to improve the probability that the true tool life curve existed within the prior sample paths. The prior n and C distributions were taken as a joint pdf, where the two constants were independent of each other. Random samples were drawn from the prior joint pdf of n and C and the Taylor tool life curve was calculated for each $\{n,C\}$ pair; this exercise was repeated 1×10^5 times. The cutting speed was calculated using $V = \pi d\Omega$, where d is the tool diameter (19.05 mm for this study) and Ω is the spindle speed in rev/min (a range of 1500 rpm to 7500 rpm was selected). The prior probability that any sample paths is the true tool life curve for this case is 1×10^{-5}. The collection of prior sample paths could then be used to determine the cumulative density function (cdf) of tool life at any spindle speed in the domain.

To demonstrate the approach, consider a scenario where the $\{n,C\}$ values can take only 10 different combinations (see Table 1). For the prior, it is assumed that any combination is equally likely to be the true combination. This gives a probability of 0.1 for each $\{n,C\}$ pair since there are 10 possible pairs. The Taylor tool life values are calculated for all spindle speeds in the domain for the 10 $\{n,C\}$ pairs. Figure 1 shows the 10 tool life curves. These are the sample paths or random walks, each generated using a different $\{n,C\}$ sample. Table 1 includes the tool life values for each $\{n,C\}$ sample at 2500 rpm, 5000 rpm, and 7500 rpm. Figure 2 displays the discrete tool life cdf at the three spindle speeds. These cdfs give the probability of tool failure as a function of tool life, $p(T)$. For example, the probability of tool failure for a required tool life of 10 min is effectively zero at 2500 rpm, it is approximately 0.9 at 5000 rpm, and 1 for 7500 rpm. These results match the trend of reduced tool life with increased cutting speed

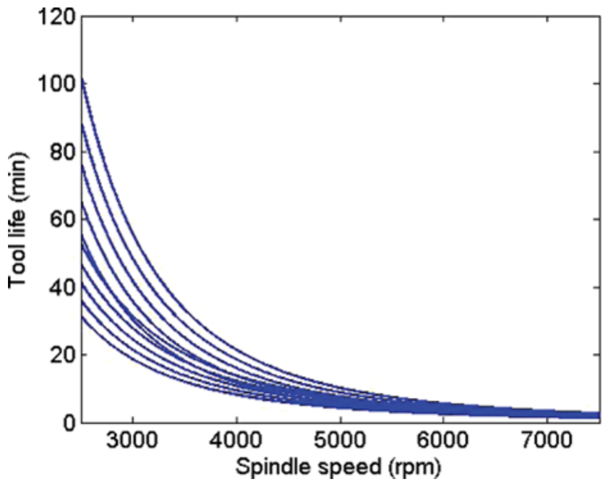

FIGURE 1 Sample tool life curves for the $\{n,C\}$ pairs listed in Table 1. (Figure available in color online.)

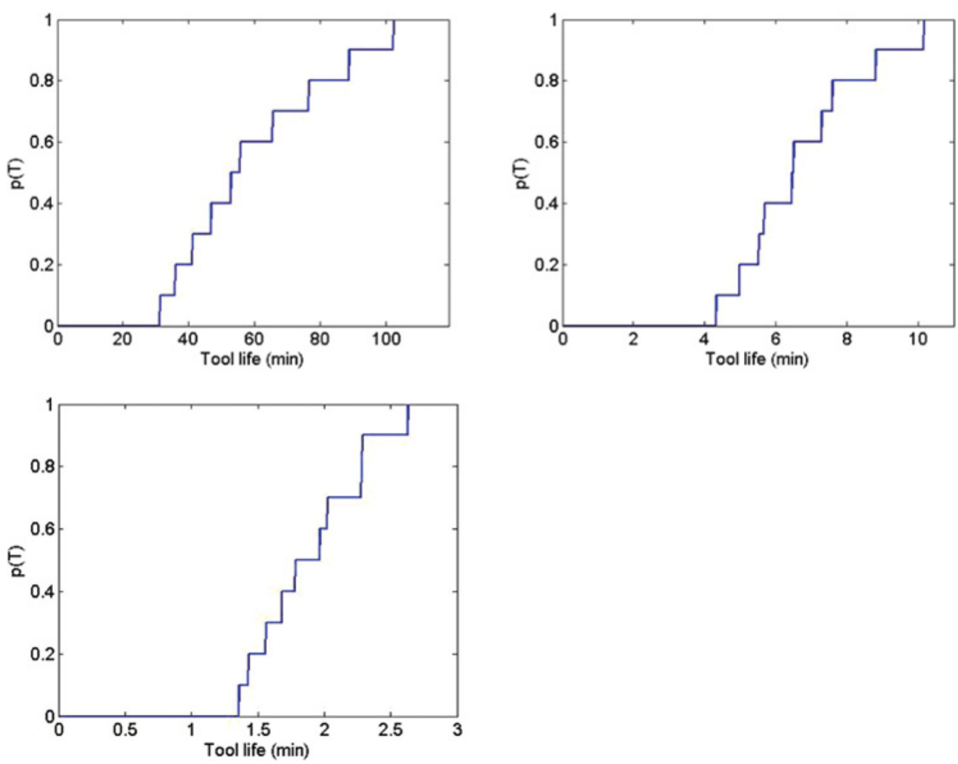

FIGURE 2 Prior cdf of tool life at 2500 rpm/149.6 m/min (top left), 5000 rpm/299.2 m/min (top right), and 7500 rpm/448.9 m/min (bottom left). (Figure available in color online.)

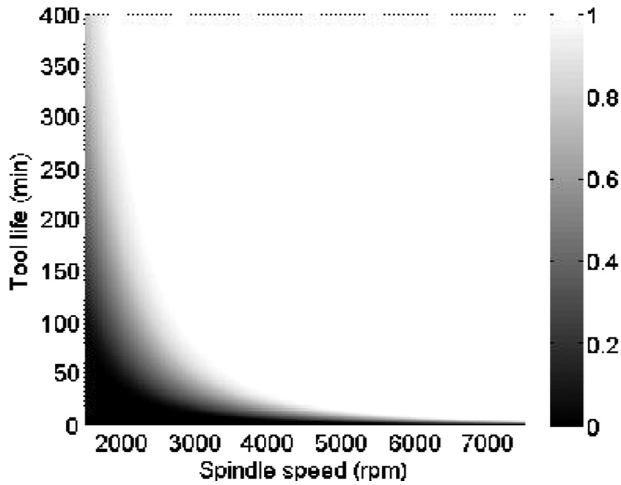

FIGURE 3 Prior cumulative distribution of tool life as a function of spindle speed.

This procedure was completed for 1×10^5 sample paths that were generated by drawing random samples from the prior joint $\{n, C\}$ distribution. Figure 3 shows the prior cumulative distribution of tool life as a function of spindle speed. The color bar gives the probability of tool failure at a selected tool life for any spindle speed in the domain. As expected, the probability of failure increases with spindle speed for a particular tool life value.

Likelihood Function

Tool life is generally considered to be stochastic in nature. If a tool life experiment is repeated under the same conditions, it is unlikely that exactly the same tool life would be obtained over multiple trials. The likelihood function is designed to account for this behavior. To illustrate, consider that a tool life of 55.8 min was obtained at 2500 rpm. The user might believe that a tool life between 45 min and 65 min is therefore very likely if the experiment was to be repeated. The user may also believe that it is not very likely that the tool will last less than 35 min or greater than 75 min based on the initial result. This information is taken into account using the likelihood function provided in Equation (4):

$$l = e^{-\frac{(T-T_m)^2}{k}} \tag{4}$$

where l is the likelihood function, T_m is the measured tool life, T is the tool life value for a sample curve at the experimental spindle speed, and k depends on the tool life distribution. The likelihood function is expressed as a non-normalized normal distribution, where $k = 2\sigma^2$ and σ is the standard deviation of tool life.

This likelihood function describes how likely it is that that the sample tool life curve is the correct curve given the measurement result at a particular spindle speed. If the tool life curve value is near the measurement result, then the likelihood value is high. Otherwise, it is low. The likelihood function defined in Equation (4) does not completely reject paths which differ significantly from the experimental result; it simply yields a small value for these paths. To illustrate, again consider the 10 possible $\{n, C\}$ pairs listed in Table 1. Assume an experimental tool life of 55.8 min was obtained at 2500 rpm. At 2500 rpm, each sample tool life curve will have a value of tool life value depending on the $\{n, C\}$ combination used to generate that sample path. The likelihood function can be interpreted as assigning weights to sample paths from zero to unity, where zero indicates that the selected combination is not likely at all and unity identifies the most likely combination. The likelihood for each sample tool life curve was calculated using Equation (4) with a measured tool life of 55.8 min. The parameter T in the equation is the tool life at the experimental spindle speed (in this example, 2500 rpm) for a particular sample tool life curve. The value of k is selected by the user based on his/her beliefs about the experimental uncertainty. For this study, the standard deviation for an experimental result was assumed to be 20% of the measured value. Table 2 lists the likelihood values for each possible $\{n, C\}$ pair. The likelihood values listed in Table 2 imply that {0.30, 500} is most likely to be the correct $\{n, C\}$ combination, whereas {0.30, 600} is the least likely.

TABLE 2 Likelihood Probabilities for Sample $\{n, C\}$ Pairs Given Experimental Tool Life of 55.8 min at 2500 rpm/149.6 m/min

		Tool life (min)				
Sample	$\{n, C\}$	2500 rpm 149.6 m/min	5000 rpm 299.2 m/min	7500 rpm 448.9 m/min	Prior	Likelihood
1	{0.30, 500}	55.8	5.5	1.4	0.10	1.000
2	{0.30, 525}	65.7	6.5	1.7	0.10	0.677
3	{0.30, 550}	76.7	7.6	2.0	0.10	0.174
4	{0.30, 575}	88.9	8.8	2.3	0.10	0.012
5	{0.30, 600}	102.5	10.2	2.6	0.10	0.000
6	{0.35, 500}	31.4	4.3	1.4	0.10	0.092
7	{0.35, 525}	36.1	5.0	1.6	0.10	0.211
8	(0.35, 550}	41.2	5.7	1.8	0.10	0.427
9	{0.35, 575}	46.8	6.5	2.0	0.10	0.724
10	(0.35, 600}	52.9	7.3	2.3	0.10	0.966

The likelihood values are rounded to three significant digits.

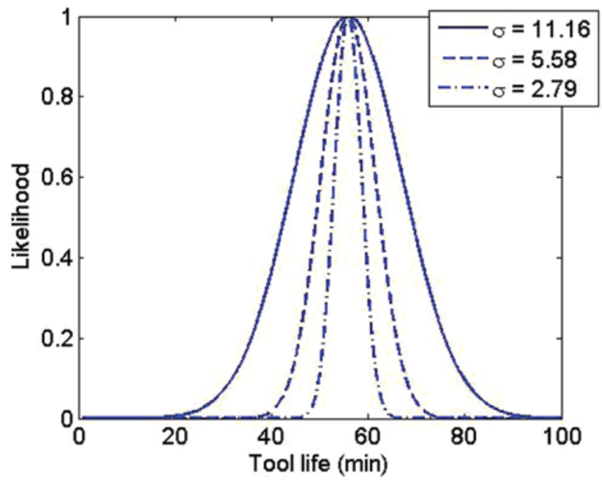

FIGURE 4 Likelihood for various uncertainty levels based on a measured tool life of 55.8 min at 2500 rpm/149.6 m/min. (Figure available in color online.)

Figure 4 shows the likelihood function for $T_m = 55.8$ min at 2500 rpm for different σ values (and, therefore, k values). As seen in the figure, increased uncertainty (higher σ) widens the likelihood function so that comparatively higher weights are assigned to sample curves far from the experimental result. Subsequently, larger uncertainty yields a more conservative estimate of tool life. Although the value of σ is considered constant in this study, it could also be expressed as a function of spindle speed.

Bayesian Updating

As noted, the likelihood function [Equation (4)] describes how likely it is that that the sample tool life curve is the correct curve given the measurement result at a particular spindle speed. The prior probability for each path is $1/N$, where N is the number of sample paths. According to Bayes' rule, the posterior distribution is obtained from the product of the prior and the likelihood. The posterior probability for each path is then normalized so that the sum is equal to unity (see Table 3).

At each spindle speed, the updated probabilities of sample tool life curves provide an updated distribution of tool life. Thus, a tool life experiment at any spindle speed updates the tool life distribution at all spindle speeds. Figure 5 displays updated posterior distributions at 2500 rpm, 5000 rpm, and 7500 rpm given an experimental result of $T_m = 55.8$ min at 2500 rpm. Figure 5 also shows the prior tool life cdfs for comparison. For the posterior cdf calculation, the updated probabilities, or weights, of the sample paths must be considered.

TABLE 3 Posterior Probabilities for Sample $\{n,C\}$ Pairs after the First Update

Sample	$\{n,C\}$	Prior	Likelihood	Posterior (non-normalized)	Posterior (normalized)
1	{0.30, 500}	0.10	1.000	0.100	0.233
2	{0.30, 525}	0.10	0.677	0.068	0.158
3	(0.30, 550}	0.10	0.174	0.017	0.041
4	{0.30, 575}	0.10	0.012	0.001	0.003
5	(0.30, 600}	0.10	0.000	0.000	0.000
6	{0.35, 500}	0.10	0.092	0.009	0.021
7	{0.35, 525}	0.10	0.211	0.021	0.049
8	(0.35, 550}	0.10	0.427	0.043	0.100
9	{0.35, 575}	0.10	0.724	0.072	0.169
10	(0.35, 600}	0.10	0.966	0.097	0.226
				$\Sigma = 0.428$	$\Sigma = 1.00$

For multiple experimental results, the posterior after the first update becomes the prior for the second update and so on. For example, consider a second experimental tool life of 5 min at 5000 rpm. The posterior probabilities of the sample paths shown in Table 3 are now the prior probabilities for the second update. The updating procedure is repeated to obtain

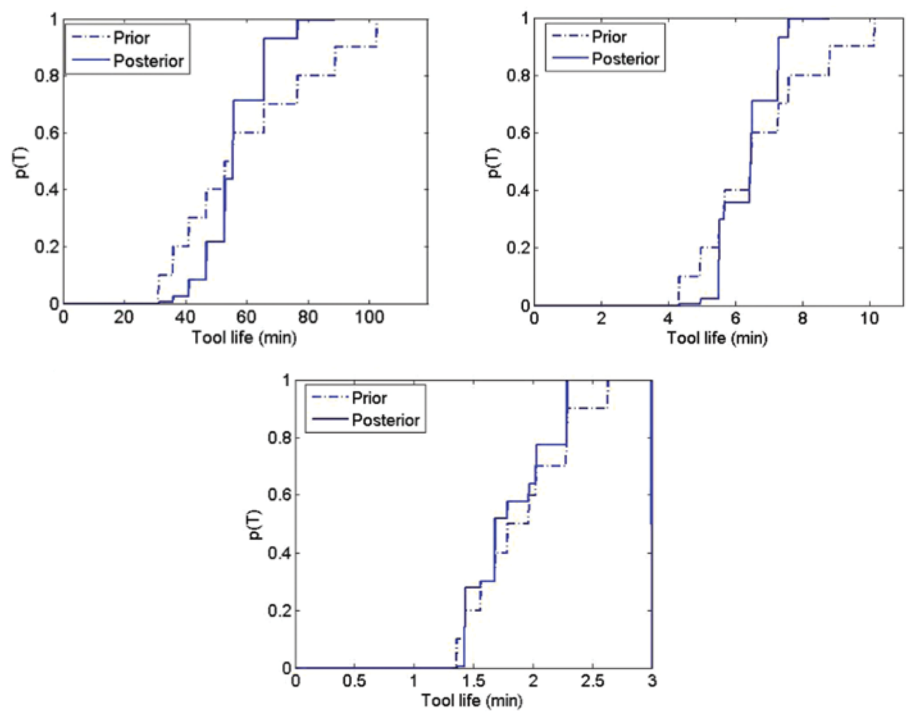

FIGURE 5 Posterior and prior tool life cdfs at 2500 rpm/149.6 m/min (top left), 5000 rpm/299.2 m/min (top right), and 7500 rpm/448.9 m/min (bottom left). (Figure available in color online.)

TABLE 4 Likelihood for Sample $\{n,C\}$ Pairs Given an Experimental Tool Life of 5 min at 5000 rpm/299.2 m/min

Sample	$\{n,C\}$	Tool life (min) 2500 rpm 149.6 m/min	5000 rpm 299.2 m/min	7500 rpm 448.9 m/min	Prior	Likelihood
1	{0.30, 500}	55.8	5.5	1.4	0.233	0.866
2	{0.30, 525}	65.7	6.5	1.7	0.158	0.318
3	(0.30, 550}	76.7	7.6	2.0	0.041	0.034
4	{0.30, 575}	88.9	8.8	2.3	0.003	0.001
5	(0.30, 600}	102.5	10.2	2.6	0.000	0.000
6	{0.35, 500}	31.4	4.3	1.4	0.021	0.802
7	{0.35, 525}	36.1	5.0	1.6	0.049	1.000
8	(0.35, 550}	41.2	5.7	1.8	0.100	0.787
9	{0.35, 575}	46.8	6.5	2.0	0.169	0.343
10	(0.35, 600}	52.9	7.3	2.3	0.226	0.071

the posterior probabilities of the sample pairs (see Tables 4 and 5). Figure 6 displays updated posterior distribution at 2500 rpm, 5000 rpm, and 7500 rpm after the second update. The mean, standard deviation, and correlation coefficient can be determined from the posterior probabilities using the following relations.

$$\mu_n = \sum nP(n) \tag{5a}$$

$$\mu_C = \sum CP(C) \tag{5b}$$

$$\sigma_n = \sum (n - \mu_n)^2 P(n) \tag{5c}$$

$$\sigma_C = \sum (C - \mu_C)^2 P(C) \tag{5d}$$

TABLE 5 Posterior Probabilities for Sample $\{n,C\}$ Pairs after the Second Update

Sample	$\{n,C\}$	Prior	Likelihood	Posterior (non-normalized)	Posterior (normalized)
1	{0.30, 500}	0.233	0.866	0.202	0.428
2	{0.30, 525}	0.158	0.318	0.050	0.106
3	(0.30, 550}	0.041	0.034	0.001	0.003
4	{0.30, 575}	0.003	0.001	0.000	0.000
5	(0.30, 600}	0.000	0.000	0.000	0.000
6	{0.35, 500}	0.021	0.802	0.017	0.036
7	{0.35, 525}	0.049	1.000	0.049	0.104
8	(0.35, 550}	0.100	0.787	0.078	0.166
9	{0.35, 575}	0.169	0.343	0.058	0.123
10	(0.35, 600}	0.226	0.071	0.016	0.034
				$\Sigma = 0.471$	$\Sigma = 1.00$

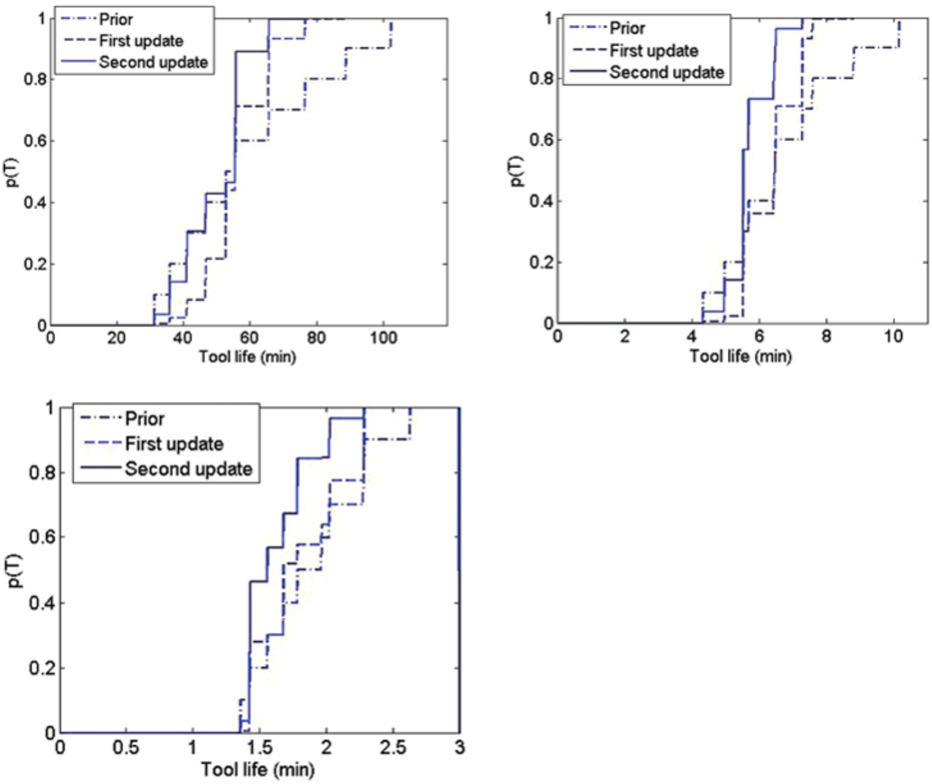

FIGURE 6 Posterior and prior tool life cdfs at 2500 rpm/149.6 m/min (top left), 5000 rpm/299.2 m/min (top right), and 7500 rpm/448.9 m/min (bottom left). (Figure available in color online.)

$$\rho_{n,C} = \frac{\sum nCP(n, C) - \mu_n \mu_C}{\sigma_n \sigma_C} \tag{5e}$$

In these equations, the summations are carried out over all N samples; $P(n)$, $P(C)$, and $P(n,C)$ are the posterior probabilities for n, C, and the $\{n,C\}$ pairs, respectively; μ_n and μ_C are the mean values of n and C, respectively; σ_n and σ_C are the standard deviations of n and C, respectively; and $\rho_{n,C}$ is the correlation coefficient between n and C.

Experimental Setup

The experimental steps followed to collect tool life data are described in this section. Down-milling tool wear tests were completed using a 19.05 mm diameter single-insert Kennametal endmill (KICR073SD30333C) on a Mikron UCP 600 Vario machining center. The workpiece material was

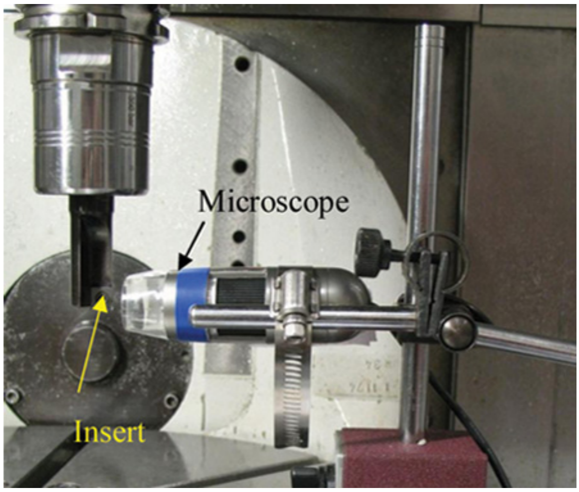

FIGURE 7 Setup for interrupted FWW measurements. (Figure available in color online.)

AISI 1018 steel. The insert was a 9.53 mm square uncoated carbide Kennametal insert (107888126 C9 JC) with zero rake and helix angles and a 15-deg relief angle.

The first test was performed at a spindle speed of 1500 rpm (V= 89.8 m/min). The feed per tooth was 0.06 mm/tooth and the axial and radial depths of cut were 3.0 mm and 4.7 mm (25% radial immersion), respectively. All tests were performed without cutting fluid. To avoid removing the insert/tool from the spindle, a portable microscope (60 × magnification) was used to record digital images of the rake and flank surfaces at regular intervals. Tool life, T, was defined as the time required for the insert to reach a maximum FWW of 0.3 mm (no crater wear was observed in these tests). Figure 7 shows the microscope setup for recording the flank surface. The calibrated digital images were then used to identify the FWW. Figure 8 shows the variation of FWW with cutting time for tests at 1500 rpm (V= 89.8 m/min). Microscopic images of the relief face for selected cutting times are displayed in Figure 9.

As seen in Figure 8, the time to reach a FWW of 0.3 mm was 255.3 min for testing at 1500 rpm (V= 89.8 m/min). Additional tests were also completed at 3750 rpm (V= 224.4 m/min) and 6250 rpm (V= 374.0 m/min). Other conditions were maintained constant and the same procedure was followed to measure tool life. The variation in FWW with cutting time for all three spindle speeds is displayed in Figure 10.

The ‘o’ symbols denote the intervals at which FWW was recorded. To establish the tool life for each test, linear interpolation was applied between adjacent measurement points if the FWW exceeded 0.3 mm for the final

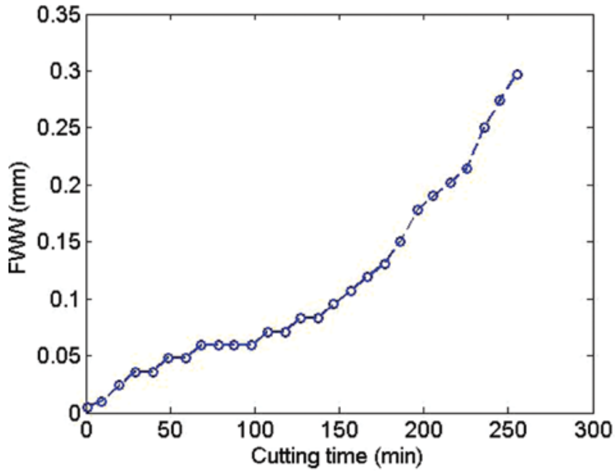

FIGURE 8 Increase in FWW with cutting time at 1500 rpm/89.8 m/min. (Figure available in color online.)

measurement interval. As expected, the tool life reduced with increasing spindle speed. These experimental results were then used to update the prior distributions of tool life, *T*, over a range of spindle speeds. Table 6 summarizes the experimental results used for updating.

Tool Life Predictions

The experimental tool life results were used to update the tool life distribution. The procedure follows.

1. For each experimental result, a likelihood value was calculated for each sample path from Equation (4). The prior probability of each sample path was 1×10^{-5}.

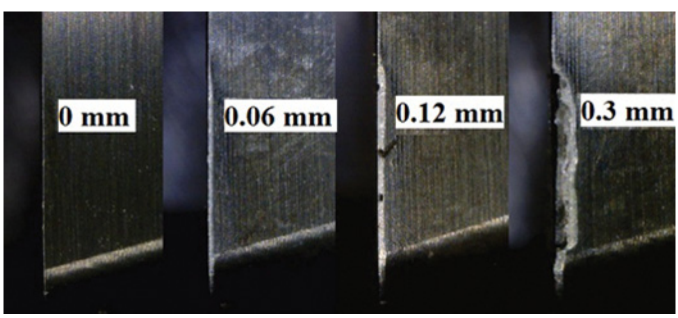

FIGURE 9 Images of FWW at 60 × magnification for 1500 rpm/89.8 m/min tests. The cutting times from left to right are {0, 78.5, 166.4, and 255.3} min. (Figure available in color online.)

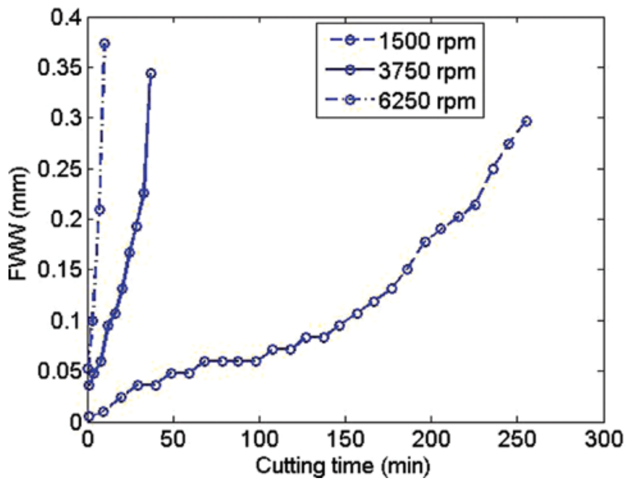

FIGURE 10 Increase in FWW with cutting time at three spindle speeds. (Figure available in color online.)

2. The posterior probability of each sample path was calculated as the product of the prior probability and the likelihood.
3. The posterior probabilities were normalized so that the sum was equal to unity.
4. For multiple experimental results, the posterior probabilities after the first update became the prior probabilities for the second and so on.

Figure 11 shows the posterior tool life cdf. The posterior probabilities of sample paths were used to determine the mean, standard deviation, and the correlation coefficient of the posterior $\{n, C\}$ distribution using Equations (5a) through (5e). The values were $\{0.342, 0.0075\}$ for n and $\{649.7, 33.74\}$for C, where the first term in the parenthesis represents the mean and the second term represents the standard deviation. The correlation coefficient between n and C was 0.67. Recall that the prior $\{n, C\}$ distribution was taken as uniform.

The posterior (updated) tool life distribution was next used to predict tool life at spindle speeds other than the ones at which the tool

TABLE 6 Experimental Tool Life Results Used for Updating

Test	Spindle speed (rpm)	Cutting speed (m/min)	Tool life (min)
1	1500	89.8	255.3
2	3750	224.4	35.5
3	6250	374.0	8.5

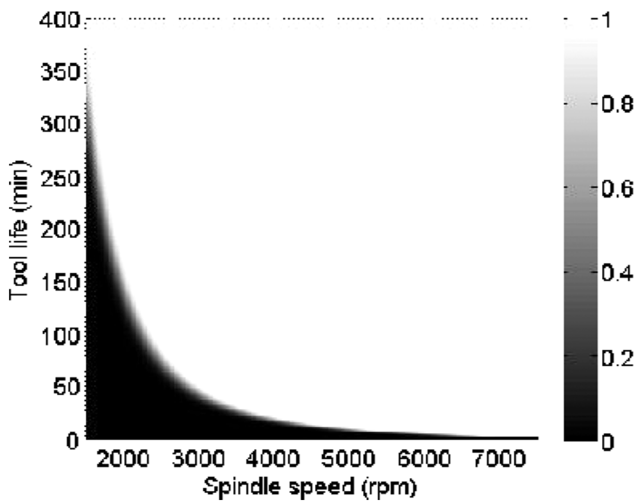

FIGURE 11 Posterior cdf of tool life.

wear experiments were performed. The posterior distribution was used to predict tool life at {2500, 5000, and 7500} rpm, which correspond to cutting speeds of {149.6, 299.2, and 448.9} m/min. Three tests were performed at each spindle speed to identify the non-repeatability. The tests were performed using the same parameters (other than spindle speed) as stated previously and the same procedure was followed to measure tool life. As before, tool life was set to be the time to reach a FWW of 0.3 mm. Table 7 shows the experimental tool life values observed from the nine tests.

The experimental tool life was compared to the predicted posterior distributions of tool life, T, at the corresponding spindle speeds. Additionally, a least squares curve fit of the Taylor tool life equation was completed using the results provided in Table 6. The values of n and C obtained from the least square fit were 0.4553 and 1120, respectively. Using this deterministic

TABLE 7 Experimental Values of Tool Life for Comparison to Predictions

Test	Spindle speed (rpm)	Cutting speed (m/min)	Tool life (min)
1	2500	149.6	50.1
2	2500	149.6	68.5
3	2500	149.6	72.0
4	5000	299.2	11.5
5	5000	299.2	9.5
6	5000	299.2	8.5
7	7500	448.8	2.6
8	7500	448.8	3.3
9	7500	448.8	3.2

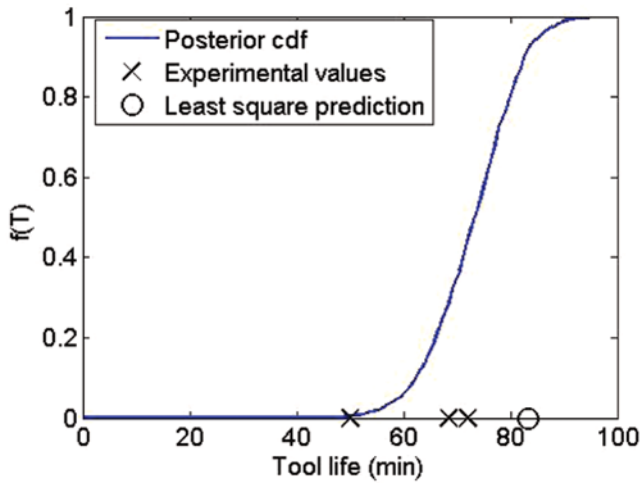

FIGURE 12 Posterior tool life cdf at 2500 rpm/149.6 m/min. The experimental data is denoted by 'x'. (Figure available in color online.)

model, the tool life values were also predicted and compared with experiment. Figures 12, 13, and 14 display the posterior distributions of tool life at 2500 rpm ($V = 149.6$ m/min), 5000 rpm ($V = 299.2$ m/min), and 7500 rpm ($V = 448.9$ m/min), respectively.

The experimental results are marked using the 'x' symbols and the least squares prediction by the 'o' symbols on the graphs. As seen from the figures, the predicted posterior distributions provide good agreement with the experimental results, while the least squares predictions are less

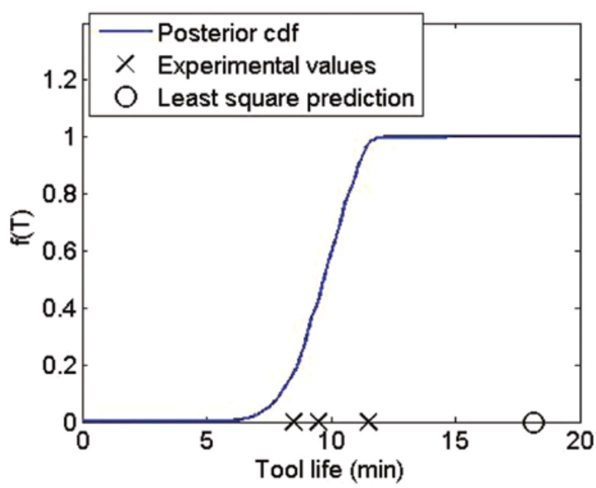

FIGURE 13 Posterior tool life cdf at 5000 rpm/299.2 m/min. (Figure available in color online.)

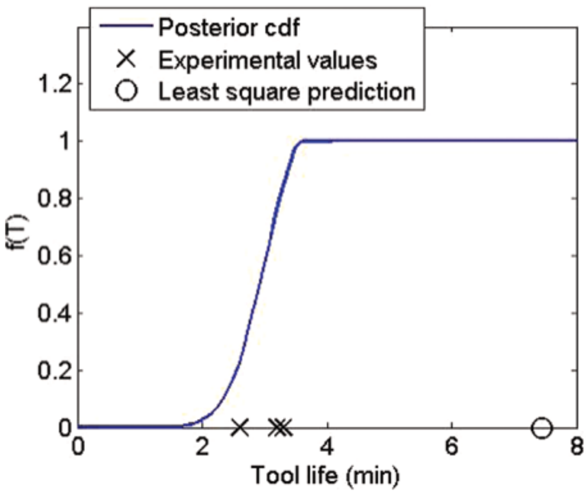

FIGURE 14 Posterior tool life cdf at 7500 rpm/448.9 m/min. (Figure available in color online.)

accurate at higher spindle speeds. Although the least squares fit was good ($R^2 = 0.9998$), the fit parameters were not reliable at higher spindle speeds. The 95% confidence bounds for n and C were determined using MATLAB's curve fitting toolbox; the values were (0.232, 0.6786) for n and (−258, 2499) for C. One explanation for the poor performance of the curve fit is that statistical curve fits generally require a large amount of data to achieve confidence in the fit parameters and extrapolation of the prediction outside the test range is often not recommended.

BAYESIAN INFERENCE OF THE TAYLOR-TYPE TOOL LIFE MODEL FOR TURNING USING THE RANDOM SURFACE METHOD

In this section, Bayesian inference of the Taylor-type tool life model [Equation (2)] using the random surface method is described. In Equation (2), there is uncertainty in the exponents, p and q, and the constant, C. As a result, there is uncertainty in the tool life. Note that tool life is dependent on both cutting speed and feed rate according to Equation (2). For given values of p, q, and C, tool life may be described using a three-dimensional surface that is a function of cutting speed and feed rate. Therefore, the tool life surface was predicted by generating N tool life (sample) surfaces, each representing the true tool life surface with equal probability. As before, the prior probability that any sample surface is the true tool life surface is $1/N$. For this case, Bayes' rule can be written as the following product.

$$p(surface = truetool\ life\ surface|\ test\ results) = \frac{p(test\ result|\ surface = true\ tool\ life\ surface)}{p(test\ result)} \cdot p(surface = true\ tool\ life\ surface)$$

Here, $p(surface = true\ tool\ life\ surface)$ is the prior probability that a given path is the true tool life surface. Also, $p(test\ result|surface = true\ tool\ life\ surface)$ is referred to as the likelihood, $p(test\ result)$ is a normalization constant, and $p(surface = true\ tool\ life\ surface|test\ result)$ is the posterior probability of the sample tool life surface given a test result. The prior sample paths were generated using random samples from a $\{p,q,C\}$ joint pdf. The prior (initial) p, q, and C distributions were selected based on a literature review.

Establishing the Prior

As noted, the first step in applying Bayesian inference is to determine the prior distribution. The cutting tool used for wear testing was a coated carbide insert and the workpiece material was a forged chrome alloy steel. The turning experiments were performed on an Okuma LC-40 CNC lathe. In this case, the prior was a joint probability distribution for the Taylor-type tool life constants, p, q, and C. The initial beliefs were:

1. in general, the value of the exponent p is greater than the exponent q due to a stronger influence of cutting speed on tool wear, but this is not a strict requirement
2. the value of p is between 2 and 6 and q is between 1.5 and 3 (Tlusty, 2000)
3. the value of C is sensitive to the values of p and q due to the nature of the tool life equation and is in the range of 1×10^6 to 1×10^8.

In this case, information was available to supply only a general range of p, q, and C. Therefore, the prior was assumed to be joint uniform distribution, i.e., it was equally likely to obtain any value within the specified range. The constants were assumed to be independent of each other for the prior. In cases where experimental data using the same tool-material combination is available, a more informative prior (such as a normal distribution) can be selected. For this study, the marginal prior pdfs of the constants were specified as:

$$p = U(2,6), q = U(1,5),\ \text{and}\ C = U(1 \times 10^6, 1 \times 10^8)$$

TABLE 8 Prior Probabilities for Sample $\{p,q,C\}$ Triplets

Sample	$\{p,q,C\}$	Tool life (min) {150 m/min, 0.5 mm/rev}	{200 m/min, 0.5 mm/rev}	{150 m/min, 0.6 mm/rev}	Prior
1	$\{2.50,2.50,1\times10^7\}$	205.3	100.0	130.1	0.10
2	$\{2.75,2.50,1\times10^7\}$	58.7	26.6	37.2	0.10
3	$\{2.50,2.25,1\times10^7\}$	172.6	84.1	114.5	0.10
4	$\{2.75,2.25,1\times10^7\}$	49.3	22.4	32.7	0.10
5	$\{2.50,2.50,5\times10^6\}$	102.6	50.0	65.1	0.10
6	$\{2.75,2.50,5\times10^6\}$	29.3	13.3	18.6	0.10
7	$\{2.50,2.25,5\times10^6\}$	86.3	42.0	57.3	0.10
8	$\{2.75,2.25,5\times10^6\}$	24.7	11.2	16.4	0.10
9	$\{2.50,2.25,7.5\times10^6\}$	129.5	63.1	85.9	0.10
10	$\{2.75,2.25,7.5\times10^6\}$	37.0	16.8	24.5	0.10

Random samples were drawn from the prior joint pdf of p, q, and C and the Taylor-type tool life surface was calculated for each $\{p,q,C\}$ triplet. In total, 1×10^5 tool life surfaces were generated. The range of cutting speed was taken from 150 m/min to 250 m/min and feed rate from 0.5 mm/rev to 0.6 mm/rev. The prior probability that any sample surface is the true tool life surface for this case is 1×10^{-5}. The updating procedure is the same as for the random walk approach. The only difference is that the random samples form three-dimensional surfaces, rather than two-dimensional paths. To demonstrate the procedure, again consider 10 possible combinations of $\{p,q,C\}$; see Table 8.

The prior assumes that any combination is equally likely to be the true combination, so each $\{p,q,C\}$ triplet was assigned a probability of 0.1. The Taylor-type tool life values were calculated for all cutting speeds and feed rates in the domain for the 10 $\{p,q,C\}$ triplets. Figure 15 shows the sample tool life surfaces. Table 8 also provides the tool life values for all $\{p,q,C\}$ combinations at cutting conditions of {150 m/min, 0.5 mm/rev}, {150 m/min, 0.6 mm/rev}, and {200 m/min, 0.5 mm/rev}. Figure 16 shows the discrete cdf of tool life at the same cutting conditions. The cdf gives the probability of tool failure as a function of tool life.

The procedure was completed for 1×10^5 sample surfaces generated by drawing random samples from the prior $\{p,q,C\}$ distribution. Since tool life depends on cutting speed as well as feed rate, the prior cdf as a function of cutting speed is conditioned on the feed rate value. Figure 17 shows the prior cdf of tool life as a function of cutting speed for a selected feed rate value. There is large uncertainty in the tool life due to the wide (uniform) prior distribution assumed for p, q, and C. The color bar in Figure 17 gives the probability of tool failure at a selected tool life for any spindle speed in the domain. From the prior distribution of p, q, and C, approximately 50% of the tool life values are less than 1 min and 15% are more than 400 min at

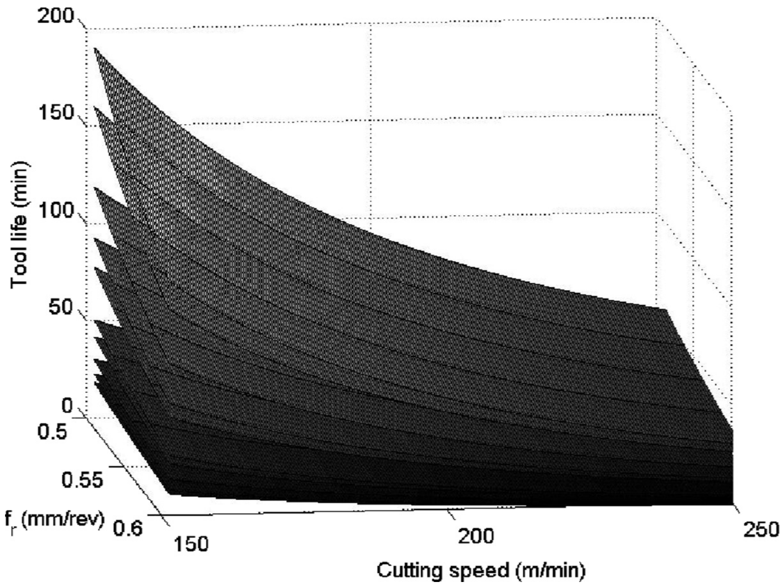

FIGURE 15 Sample tool life surfaces for $\{p,q,C\}$ triplets provided in Table 8.

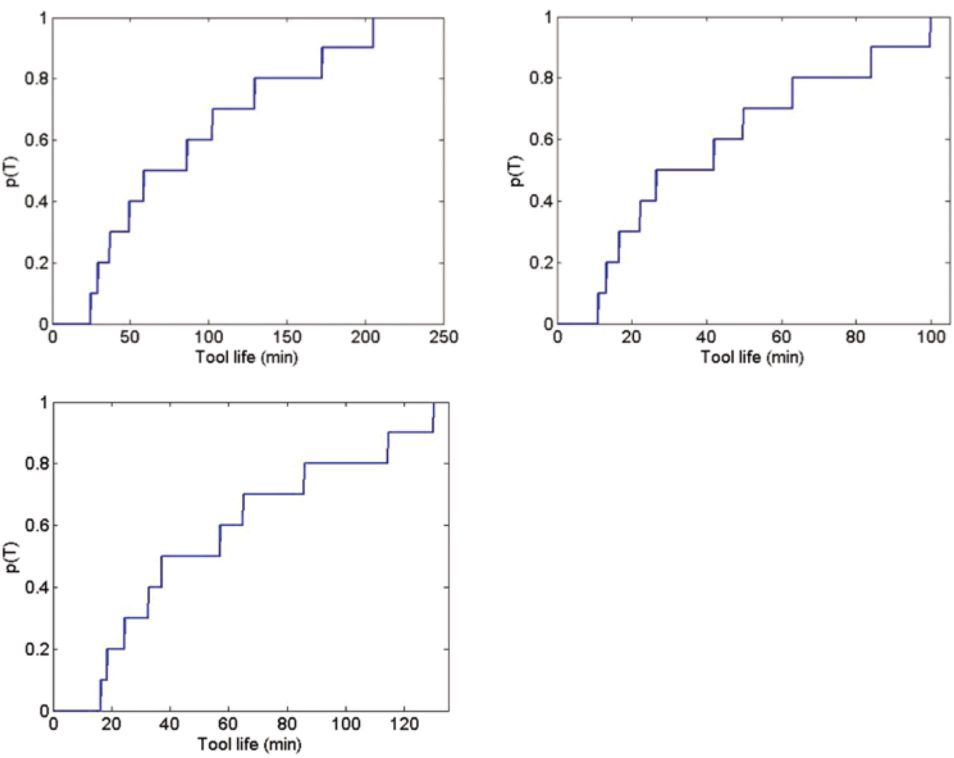

FIGURE 16 Prior cdf of tool life at {150 m/min, 0.5 mm/rev} (top left), {200 m/min, 0.5 mm/rev} (top right), and {150 m/min, 0.6 mm/rev} (bottom left). (Figure available in color online.)

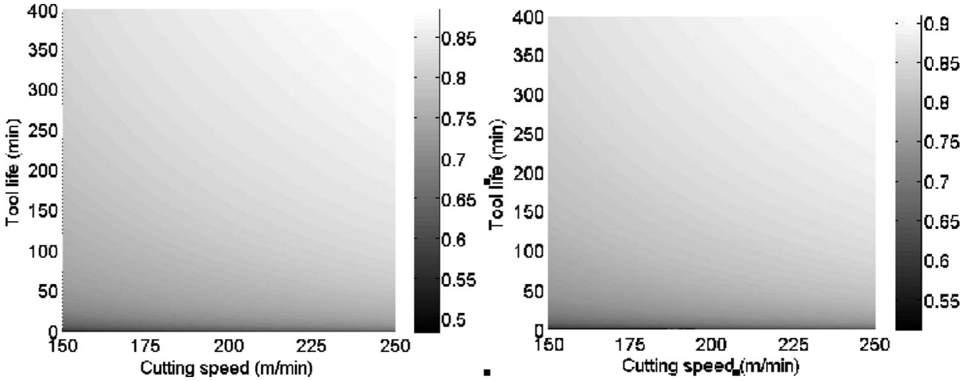

FIGURE 17 Prior cdf of tool life at 0.5 mm/rev (left) and 0.6 mm/rev (right). Note that the reduced tool life for the 0.6 mm/rev feed results in approximately 10% of the values being more than 400 min as compared to 15% at 0.5 mm/rev. The color bar scaling, therefore, differs.

{150 m/min, 0.5 mm/rev}; therefore, the probability of tool failure in Figure 17 is scaled from 0.5 to 0.85 for plotting purposes.

Likelihood Function and Bayesian Updating

The likelihood function describes how likely it is that that the sample tool life surface is the correct surface given a measurement result at a particular cutting speed and feed. As noted, the likelihood function can be interpreted as assigning weights to sample surfaces from zero to unity, where zero indicates that the selected combination is not likely at all and unity identifies the most likely combination. The likelihood function defined in Equation (4) was applied. To illustrate, again consider the 10 possible $\{p,q,C\}$ triples listed in Table 8. Assume that an experimental tool life of 102.6 min was obtained at {150 m/min, 0.5 mm/rev} and a tool life of 42.0 min was obtained at {200 m/min, 0.5 mm/rev}. A likelihood value for each sample tool life surface was calculated using Equation (4). The posterior probability calculations are shown in Tables 9 and 10. Note that these posterior probabilities are normalized and have been rounded to three significant digits. Figure 18 shows the posterior and prior cdf at {150 m/min, 0.5 mm/rev}, {200 m/min, 0.5 mm/rev}, and {150 m/min, 0.6 mm/rev}.

Experimental Setup and Results

This section describes the experimental steps following to collect turning tool life data. The cutting tool used for wear testing was a coated carbide insert (Kennametal KC9110) and the workpiece material was

TABLE 9 Likelihood and Posterior Probabilities for Sample $\{p,q,C\}$ Triplets (see Table 8) Given an Experimental Tool Life of 102.6 min at {150 m/min, 0.5 mm/rev}

	Tool life (min)					
Sample	{150 m/min, 0.5 mm/rev}	{200 m/min, 0.5 mm/rev}	{150 m/min, 0.6 mm/rev}	Prior	Likelihood	Posterior
1	205.3	100.0	130.1	0.10	0.000	0.000
2	58.7	26.6	37.2	0.10	0.101	0.044
3	172.6	84.1	114.5	0.10	0.003	0.001
4	49.3	22.4	32.7	0.10	0.034	0.015
5	102.6	50.0	65.1	0.10	1.000	0.435
6	29.3	13.3	18.6	0.10	0.002	0.001
7	86.3	42.0	57.3	0.10	0.730	0.317
8	24.7	11.2	16.4	0.10	0.001	0.000
9	129.5	63.1	85.9	0.10	0.424	0.184
10	37.0	16.8	24.5	0.10	0.006	0.003
						$\Sigma = 1$

forged *AISI 4137* chrome alloy steel. The initial outer diameter of the steel workpiece was 174.6 mm. The spindle speed was varied to maintain constant cutting speed with reducing workpiece diameter as additional cuts were completed. The depth of cut was 4.1 mm and the length of cut for a single pass was 139.7 mm with a chamfer of 63.4 deg at the end of each cut. All tests were performed without cutting fluid.

The flank and rake surfaces were recorded using a portable digital microscope ($60\times$ magnification) without removing the insert from the tool holder during the wear testing. The wear status of the tool was recorded after each pass and the calibrated digital images were used to identify the

TABLE 10 Likelihood and Posterior Probabilities for Sample $\{p,q,C\}$ Pairs (see Table 8) Given an Experimental Tool Life of 42.0 min at {200 m/min, 0.5 mm/rev}

	Tool life (min)					
Sample	{150 m/min, 0.5 mm/rev}	{200 m/min, 0.5 mm/rev}	{150 m/min, 0.6 mm/rev}	Prior	Likelihood	Posterior
1	205.3	100.0	130.1	0.000	0.000	0.000
2	58.7	26.6	37.2	0.044	0.186	0.013
3	172.6	84.1	114.5	0.001	0.000	0.000
4	49.3	22.4	32.7	0.015	0.065	0.002
5	102.6	50.0	65.1	0.435	0.635	0.452
6	29.3	13.3	18.6	0.001	0.003	0.000
7	86.3	42.0	57.3	0.317	1.000	0.520
8	24.7	11.2	16.4	0.000	0.001	0.000
9	129.5	63.1	85.9	0.184	0.043	0.013
10	37.0	16.8	24.5	0.003	0.011	0.000
						$\Sigma = 1$

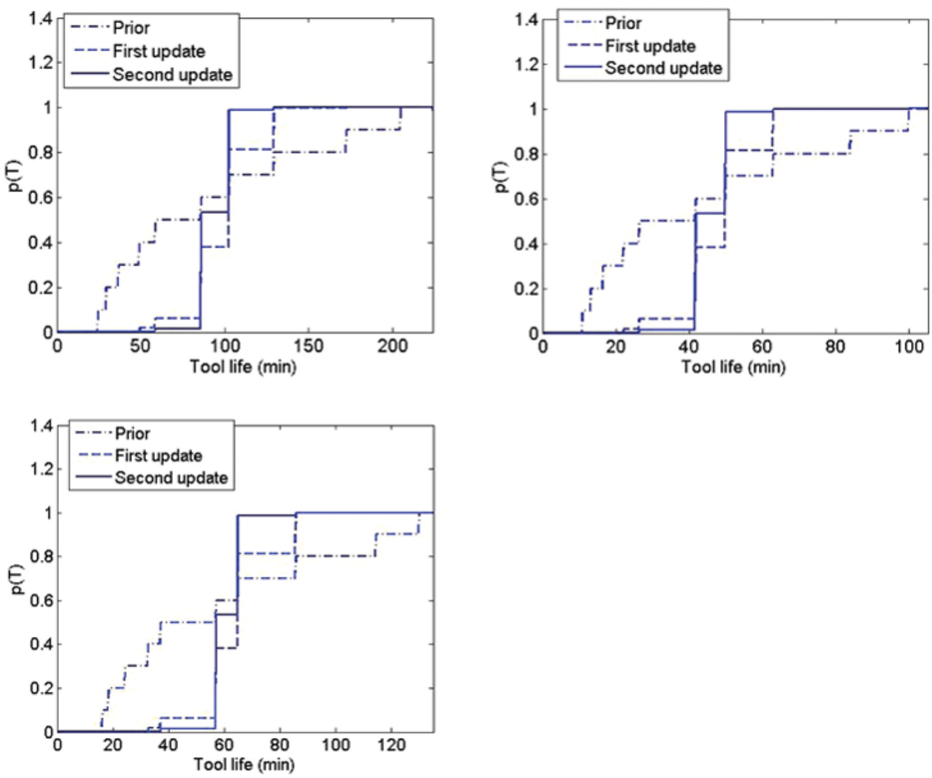

FIGURE 18 Posterior and prior cdf of tool life at {150 m/min, 0.5 mm/rev} (top left), {200 m/min, 0.5 mm/rev} (top right), and {150 m/min, 0.6 mm/rev} (bottom left). (Figure available in color online.)

flank wear width (FWW). Tool life was defined as the time required for the maximum FWW to reach 0.4 mm. The first test was completed using a cutting speed of 153.6 m/min and a feed per revolution of 0.51 mm/rev. Figure 19 shows the variation of FWW with cutting time. The time to reach a FWW of 0.4 mm was 22.5 min. Figure 20 shows images of the relief face at selected cutting times.

Two additional tests were performed at $\{V = 192.0\,\text{m/min}, f_r = 0.61\,\text{mm/rev}\}$ and $\{V = 230.4\,\text{m/min}, f_r = 0.51\,\text{mm/rev}\}$. Figure 21 shows the growth in FWW for all three test conditions. The 'o' symbols denote the intervals at which the FWW was recorded. The tool life was determined by linear interpolation between adjacent intervals if it exceeded 0.4 mm at the final measurement interval. The results of the three tests are summarized in Table 11. As expected, tool life reduced with increased cutting speed and feed rate.

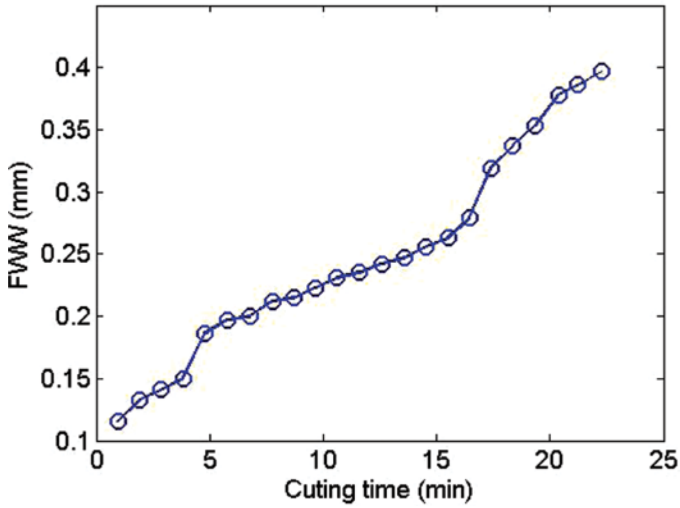

FIGURE 19 Increase in FWW with cutting time at $V = 153.6\,\text{m/min}$ and $f_r = 0.51\,\text{mm/rev}$. (Figure available in color online.)

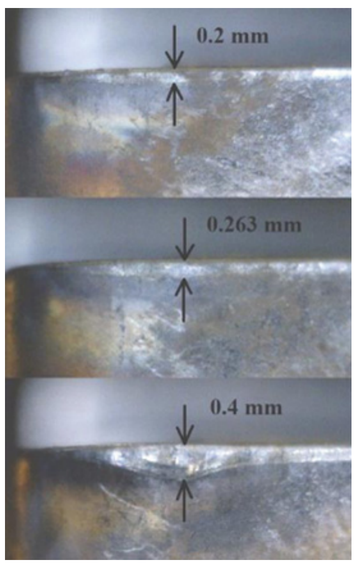

FIGURE 20 Images of FWW at 60 × magnification. The cutting times from top to bottom are {6.8, 15.5, and 22.4} min. (Figure available in color online.)

Tool Life Predictions

The experimental tool life results were used to update the prior tool life distribution using the random surface method. As noted, 1×10^5 sample surfaces were generated by sampling from the prior $\{p,q,C\}$ distribution.

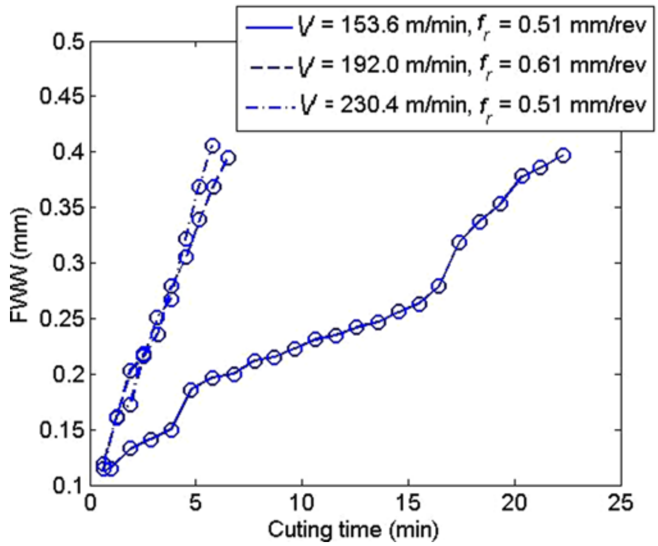

FIGURE 21 Variation of FWW with cutting time at various test conditions. (Figure available in color online.)

The likelihood for each test result was calculated using Equation (4). The prior probabilities of the sample surfaces were updated using the experimental result following the procedure described earlier. The posterior probability of each sample surface was calculated as the product of the prior probability and the likelihood. The posterior probabilities were normalized so that the sum was equal to unity.

Figure 22 shows the posterior cumulative distribution of tool life as a function of cutting speed conditioned on the feed rate value. The mean and standard deviation, for the posterior $\{p, q, C\}$ distributions were $\{3.25, 0.19\}$ for p, $\{2.81, 0.99\}$ for q, and $\{5.2 \times 10^7, 2.85 \times 10^7\}$ m/min for C, where the first term is the mean and the second is the standard deviation. The correlation coefficients were 0.64 between p and q, 0.71 between p and C, and 0.032 between q and C.

The posterior tool life distributions can be used to predict tool life at cutting conditions other than the ones at which the tool wear experiments were performed. The posterior distribution was used to predict tool life for

TABLE 11 Experimental Tool Life Results Used for Updating

Test	Cutting speed (m/min)	Feed (mm/rev)	Tool life (min)
1	153.6	0.51	22.5
2	192.0	0.61	6.5
3	230.4	0.51	5.6

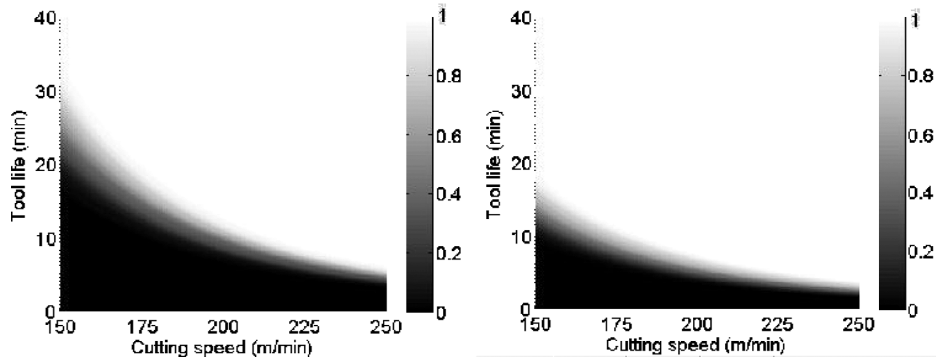

FIGURE 22 Posterior cdf of tool life at 0.5 mm/rev (left) and 0.6 mm/rev (right).

two new test conditions: {V= 192.0/min, f_r= 0.51 mm/rev} and {V= 230.4 m/min, f_r= 0.61 mm/rev}. Two tests were performed for each parameter combination. Other conditions were maintained constant and the same procedure was followed to measure tool life. As before, tool life was selected to be the time for the tool to reach a FWW of 0.4 mm. Table 12 shows the experimental tool life values observed from the four additional tests.

The deterministic Taylor-type tool life constants were calculated using a least squares best fit to the experimental tool life data listed in Table 11. The p, q, and C values were 3.39, 2.63, and 9.83×10^7 m/min, respectively. Figure 23 shows the experimental values at V= 192.0 m/min and f_r= 0.51 mm/rev (denoted by 'x'), the posterior distribution after updating, and the deterministic tool life predictions (denoted by 'o'). Figure 24 shows the results for V= 230.4 m/min and f_r= 0.61 mm/rev. As seen in Figure 23, the observed tool life values agree with both the predicted posterior distribution and the deterministic predictions at {192.0 m/min, f_r= 0.51 mm/rev}. In Figure 24, both predictions slightly overestimate the tool life. A significant difference between the two techniques, however, is that Bayesian inference assigns a probability distribution to tool life, while deterministic methods (such as curve fitting) predict a single value.

TABLE 12 Experimental Values of Tool Life for Additional Turning Tests

Test #	Cutting speed (m/min)	Feed (mm/rev)	Tool life (min)
1	192.0	0.51	11.5
2	192.0	0.51	10.3
3	230.4	0.61	2.2
4	230.4	0.61	2.6

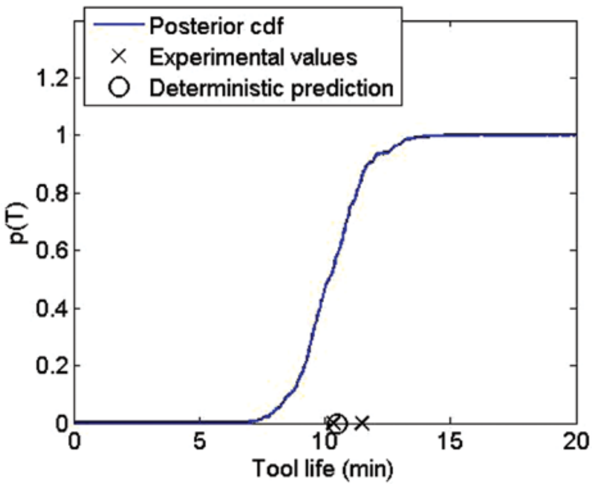

FIGURE 23 Posterior tool life cdf at {192.0 m/min, $f_r = 0.51$ mm/rev}. (Figure available in color online.)

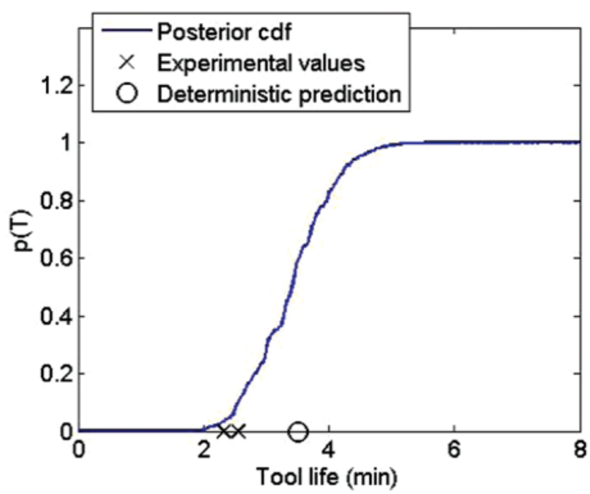

FIGURE 24 Posterior tool life cdf at {230.4 m/min, $f_r = 0.61$ mm/rev}. (Figure available in color online.)

DISCUSSION

In Bayesian inference, the posterior probability is the product of the prior and the likelihood distributions. Clearly, the posterior probabilities of the random sample paths/surfaces depend on the selection of the prior and the likelihood distributions. In this section, the influence of the prior distribution and likelihood uncertainty on the posterior is evaluated. First,

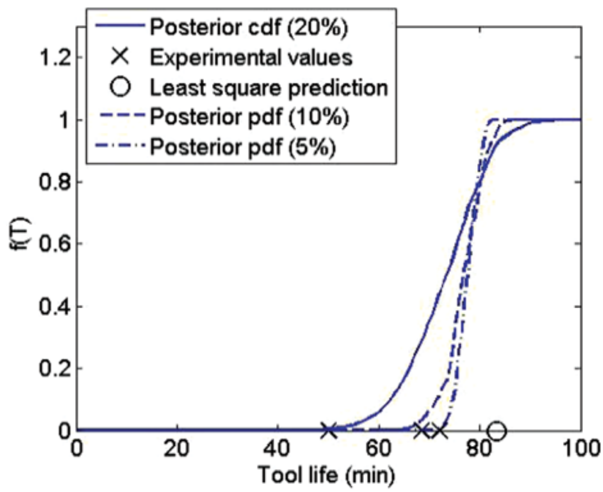

FIGURE 25 Posterior cdf at 2500 rpm for different likelihood uncertainties assumed. (Figure available in color online.)

the influence of the likelihood uncertainty on the posterior tool life cdf is examined. In the initial analysis, a likelihood uncertainty of 20% of the experimental tool life was assumed. Here, Bayesian updating was repeated for both the milling and turning tool life models using likelihood uncertainties of 5% and 10%. Figure 25 displays the milling posterior tool life predictions at 2500 rpm for different likelihood uncertainties. The experimental values are denoted as 'x' and the deterministic prediction as 'o'.

Table 13 lists the mean, standard deviation, and the correlation coefficient for the corresponding posterior n and C distributions. As seen from Figure 25 and the standard deviation values listed in Table 13, the likelihood uncertainty affects the spread of the posterior tool life distribution. A smaller likelihood uncertainty narrows the distribution. Also, as the likelihood uncertainty tends to zero, the posterior tool life cdf approaches the least squares prediction. Similar results are obtained for the turning tool life model as shown in Figure 26.

TABLE 13 Posterior $\{n, C\}$ Distribution for Different Likelihood Uncertainties in Milling

	Likelihood uncertainty		
Parameters	20%	10%	5%
$\{\mu_n, \sigma_n\}$	$\{0.34, 0.0075\}$	$\{0.35, 0.0023\}$	$\{0.35, 0.0006\}$
$\{\mu_C, \sigma_C\}$	$\{649.7, 33.7\}$	$\{676.3, 14.1\}$	$\{685.2, 7.1\}$
$\rho_{n,C}$	0.67	0.45	0.28

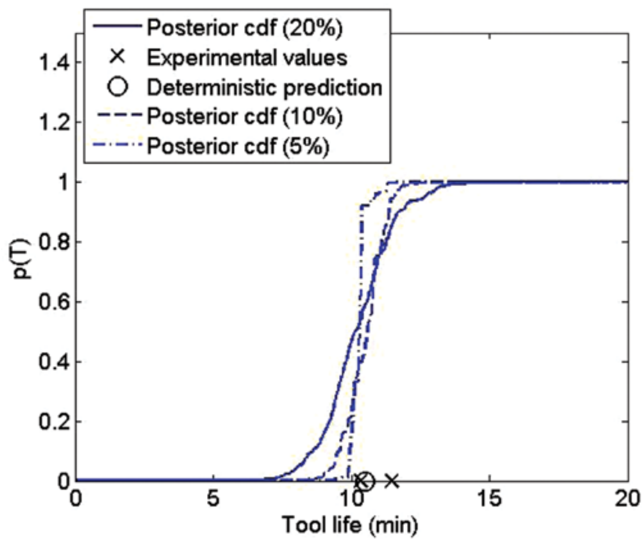

FIGURE 26 Posterior tool life cdf at {192.02 m/min, $f_r = 0.51$ mm/rev} for different likelihood uncertainties. (Figure available in color online.)

The influence of the prior distribution on the posterior distribution was also examined. As stated earlier a uniform prior was selected for $\{n, C\}$ in the milling model to generate the random sample paths. To evaluate the influence of the prior distribution, a normal distribution was selected for the prior $\{n, C\}$ distribution. The distribution was selected to be:

$$n = N(0.3, 0.03) \text{ and } C = N(250, 50)$$

where N denotes a normal distribution and the values in the parentheses identify the mean and standard deviation, respectively. The n and C random samples were assumed to be independent. The mean values of the normal $\{n, C\}$ prior distribution were deliberately selected to be lower than the posterior mean values determined earlier. Random samples were drawn from the distribution and the sample tool life curves were calculated for each. Figure 27 shows the new (normal) prior distribution. Bayesian updating was completed using the experimental results listed in Table 6.

Figure 28 shows the posterior tool life cdf for the normal prior. The posterior cdf for the uniform prior is also included for comparison. Table 14 compares the posterior means, standard deviations, and correlation coefficients for the two priors. As seen in Figure 28, the posterior tool life prediction is more conservative for the normal $\{n, C\}$ prior, which may be a preferred result in machining operations where a tool failure can lead to significant expense and lost time. A normal prior represents a more

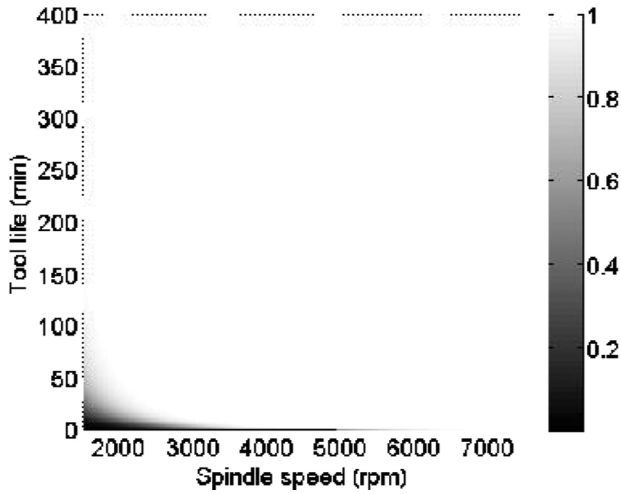

FIGURE 27 Prior cumulative distribution of tool life for the normal $\{n,C\}$ prior.

informative case where knowledge of the distribution in $\{n,C\}$ values is available. A more informative prior reflects the most likely values. Because the prior represents the initial degree of belief about the constants, if the initial belief is far from the true value, then the final results are affected. In general, the prior should be chosen to be as informative as possible considering all the available information. If enough data or prior knowledge is not available, a uniform prior may be selected.

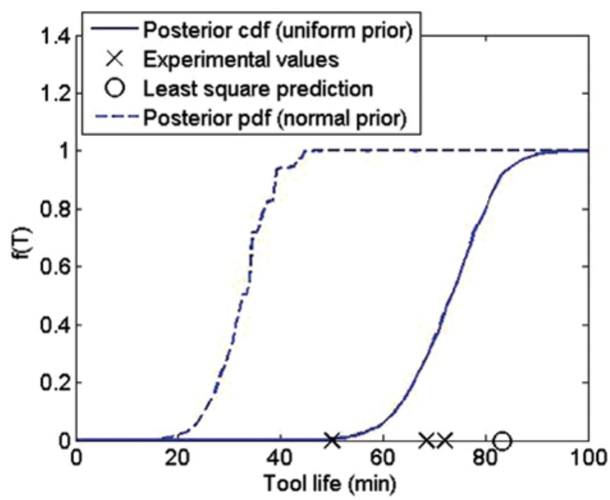

FIGURE 28 Posterior tool life cdf at 2500 rpm for different priors. (Figure available in color online.)

TABLE 14 Posterior $\{n, C\}$ Distribution for Normal and Uniform Prior

	Prior	
Parameters	Uniform	Normal
$\{\mu_n, \sigma_n\}$	{649.7,33.7}	{378.1,20.6}
$\{\mu_C, \sigma_C\}$	{0.342, 0.01}	{0.2661, 0.02}
$\rho_{n,C}$	0.67	0.65

CONCLUSION

A Bayesian inference approach to tool life prediction was demonstrated using a random walk/random surface method. The Taylor tool life model was applied to milling and a Taylor-type model to turning; however, these models were only selected because they are well-known. The approach can be implemented for extended versions of the Taylor-type tool life model that include axial and radial depth of cut effects, for example, or other, more comprehensive models as well. For example, Makarow's law (Astakhov, 2006), which identifies an optimum cutting temperature for a selected combination of work material and tool material, could be implemented as an alternative tool life model.

In Bayesian inference, tool life is characterized by a probability distribution and the distribution is updated when new information is available. When new information in the form of experimental results is obtained, uncertainty in the prior distribution can be reduced. Bayesian inference therefore provides a way to combine prior data with experimental values to update beliefs about uncertain variables. Using the random walk approach for milling, the prior probability of tool life was generated using sample tool life curves, where each path potentially represented the true tool life curve. The probability that each sample path represented the true Taylor tool life curve was updated using Bayesian inference.

A likelihood function was defined to describe how likely it was that that the sample tool life curve was the correct choice given the measurement result at a particular spindle speed. An uncertainty of 20% was assumed for the measured tool life. The posterior tool life distribution was then used to predict the values of tool life at different spindle speeds and the results were compared to experiment. The same procedure was repeated using an extended form of the Taylor tool life equation to incorporate the effects of both cutting speed and feed in turning. In this case, sample tool life surfaces were generated. The probability that a sample surface was the true tool life surface was updated using Bayesian inference. The posterior tool life distribution agreed with the experimental results in both cases. Comparisons were also made to deterministic predictions using a least squares best fit to identify the Taylor tool life model empirical constants.

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