Tool life prediction using Bayesian updating. Part 1: Milling tool life model using a discrete grid method

Jaydeep M. Karandikar [a], Ali E. Abbas [b], Tony L. Schmitz [c],*

[a] Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL, USA
[b] Department of Industrial and Enterprise Systems Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, USA
[c] Department of Mechanical Engineering and Engineering Science, University of North Carolina at Charlotte, Charlotte, NC, USA

ABSTRACT

According to the Taylor tool life equation, tool life reduces with increasing cutting speed following a power law. Additional factors can also be added, such as the feed rate, in Taylor-type models. Although these models are posed as deterministic equations, there is inherent uncertainty in the empirical constants and tool life is generally considered a stochastic process. In this work, Bayesian inference is applied to estimate model constants for both milling and turning operations while considering uncertainty.

In Part 1 of the paper, a Taylor tool life model for milling that uses an exponent, n, and a constant, C, is developed. Bayesian inference is applied to estimate the two model constants using a discrete grid method. Tool wear tests are performed using an uncoated carbide tool and 1018 steel work material. Test results are used to update initial beliefs about the constants and the updated beliefs are then used to predict tool life using a probability density function. In Part 2, an extended form of the Taylor tool life equation is implemented that includes the dependence on both cutting speed and feed for a turning operation. The dependence on cutting speed is quantified by an exponent, p, and the dependence on feed by an exponent, q; the model also includes a constant, C. Bayesian inference is applied to estimate these constants using the Metropolis–Hastings algorithm of the Markov Chain Monte Carlo (MCMC) approach. Turning tests are performed using a carbide tool and MS309 steel work material. The test results are again used to update initial beliefs about the Taylor tool life constants and the updated beliefs are used to predict tool life via a probability density function.

© 2013 Elsevier Inc. All rights reserved.

1. Introduction

Tool wear can impose a significant limitation on machining processes, particularly for hard-to-machine materials such as titanium and nickel-based superalloys. Taylor first defined an empirical relationship between tool life and cutting speed using a power law [1]:

$$VT^n = C, \qquad (1)$$

where V is the cutting speed in m/min, T is the tool life in minutes, and n and C are constants which depend on the tool-workpiece combination. The constant C is defined as the cutting speed required to obtain a tool life of 1 min. Tool life is typically defined as the time required to reach a predetermined flank wear width (FWW), although other wear features (such as crater depth) may also be applied depending on the nature of the tool wear. The Taylor tool life model is deterministic in nature, but uncertainty exists due

to: (1) factors that are unknown or not included in the model and (2) tool-to-tool performance variation. For these reasons, tool wear is often considered to be a stochastic and complex process and, therefore, difficult to predict.

Previous efforts to model tool wear as a stochastic process as indicated in the reference list in [2]. Vagnorius et al. calculated the optimal tool replacement time by determining the probability of the tool failing before the selected time using a tool reliability function [2]. Liu et al. derived a recursive formula to determine the cutting tool reliability. The maximum likelihood method was used to determine the unknown parameters in the reliability function [3]. Wiklund et al. applied the Bayesian approach to monitor tool wear using in-process information [4]. The method presented in this paper uses Bayesian inference to predict tool life at the process planning stage. The distribution of the Taylor tool life constants, n, and C, are updated using experimental tool life results. The updated distributions of n and C can then be used to predict tool life. The objective of the paper is to demonstrate the application of Bayesian updating to tool life prediction. The Taylor tool life model is used in this study, despite its potential limitations, because it is well-known and generally understood in the manufacturing community.

---

* Corresponding author. Tel.: +1 117046875086.
  E-mail address: tony.schmitz@uncc.edu (T.L. Schmitz).

Without loss of generality, the Bayesian updating method demonstrated in this paper can be applied to other available models [5].

Bayesian inference, which forms a normative and rational method for belief updating is applied in this work [6]. Let the prior distribution about an uncertain event, A, at a state of information, &, be $\{A|\&\}$, the likelihood of obtaining an experimental result B given that event A occurred be $\{B|A,\&\}$, and the probability of receiving experimental result B (without knowing A has occurred) be $\{B|\&\}$. Bayes' rule is used to determine the posterior belief about event A after observing the experiment results, $\{A|B,\&\}$ as shown in Eq. (2). Using Bayes' rule, information gained through experimentation can be combined with the prior prediction about the event to obtain a posterior distribution:

$$\left\{A\,\middle|\,B,\&\right\} = \frac{\left\{A\,\middle|\,\&\right\}\left\{B\,\middle|\,A,\&\right\}}{\left\{B\,\middle|\,\&\right\}} \qquad (2)$$

As seen in Eq. (1), the Taylor tool life model assigns a deterministic value to tool life for a selected cutting speed. In contrast, Bayesian inference assigns a probability distribution to the tool life value at a particular cutting speed. From a Bayesian standpoint, a variable which is uncertain is treated as a variable which is random and characterized by a probability distribution. The prior, or initial belief of the user, can be based on theoretical considerations, expert opinions, past experience, or data reported in the literature; the prior should be as chosen to be as informative as possible. The prior is represented as a probability distribution and, using Bayes' theorem, the probability distribution is updated when new information becomes available (from experiments, for example). As a result, Bayesian inference enables a model to incorporate uncertainty in terms of a probability distribution and beliefs about this distribution to be updated based on experimental results.

The remainder of the paper is organized as follows. Section 2 describes the application of Bayesian updating to Taylor tool life constants. Section 3 describes tool life prediction using the posterior, or updated, distributions of tool life. Section 4 compares Bayesian inference to the traditional least squares fit and suggests advantages of using the Bayesian approach. The influence of prior selection and the likelihood uncertainty on the tool life predictions is described in Section 5. Conclusions are provided in Section 6.

2. Application of Bayesian inference to the Taylor tool life model

In the Taylor tool life model provided in Eq. (1), there is uncertainty in the exponent, n, and the constant C. As a result, there is uncertainty in the tool life, T. This uncertainty can be represented as a joint probability distribution for n and C and, therefore, for the tool life, T. Bayes' rule (Eq. (2)) can be used to update the joint prior distribution of n and C. The updated distribution can then be used to update the distribution of tool life, T. In this case, the prior distribution from Eq. (2), $\{A|\mathscr{E}\}$, is the initial belief about n and C. Updating of these constants can be completed using experimental tool life data. Bayes' rule for this application is:

$$\left\{n,C\,\middle|\,T,\&\right\} \propto \left\{n,C\,\middle|\,\&\right\}\left\{T\,\middle|\,n,C,\&\right\}, \qquad (3)$$

where $\{n,C|\&\}$ is the prior joint distribution of n and C, $\{T|n,C,\&\}$ is the likelihood of observing an experimental result for tool life, T, given n and C, and $\{n,C|T,\&\}$ is the posterior joint distribution of n and C given an experimental result of tool life, T. Note that the denominator in Eq. (2), $\{B|\mathscr{E}\}$, acts as a normalizing constant and is not included in Eq. (3).

According to Bayes' rule, the posterior distribution is proportional to the product of the prior and the likelihood. This is the process of learning, or updating beliefs, when experimental results are available. For multiple experimental results, the posterior after
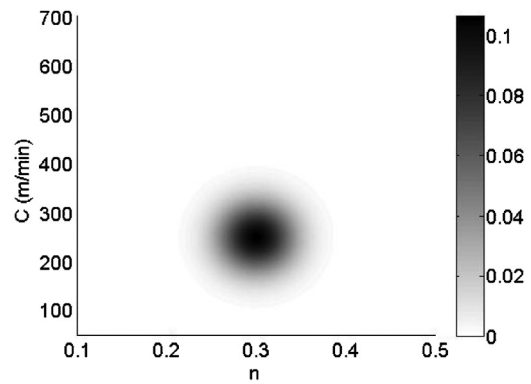

Fig. 1. Joint prior distribution of n and C. The gray scale color bar identifies the pdf value.

the first update becomes the prior for the second update and so on. Note that the posterior distributions must be normalized so that a unit area/volume under the probability density function (pdf) is obtained. This is the role of the denominator in Eq. (2).

In a milling operation, other factors such as feed rate and axial/radial depths of cut may also affect tool life, in addition to the cutting speed [7]. However, since cutting speed is often the strongest factor, Bayesian updating was performed using Eq. (1). Without loss of generality, the procedure can be applied to extended forms of the Taylor tool life equation.

2.1. Establishing the prior

The tool used in this study was an uncoated carbide (inserted) tool and the workpiece material was 1018 steel. The prior, or initial beliefs, about the tool life for this combination was therefore identified; this is the first step in applying Bayesian inference to a decision-making situation. In this case, the prior is a joint probability distribution for Taylor tool life constants, n and C. Recall that the prior can be based upon theoretical considerations, expert opinions, past experience, or data reported in the literature. A literature review was therefore completed to determine the prior joint distribution of Taylor tool life constants, n and C. Stephenson and Agapiou [8] reported the value of n to be in the range of 0.2–0.25 for uncoated carbide tools and C to be around 100 m/min for rough finishing of low carbon steels. Kronenberg [9] reported values of n and C to be in the range of 0.3–0.5 and 160–200 m/min, respectively, for machining steel with a carbide tool. Creese [10] reported typical n and C values for machining medium carbon steel with a carbide tool to be 0.32 and 240 m/min, respectively. Cui et al. [11] performed wear experiments using a carbide insert and 1018 steel workpiece. Values of n and C were reported to be 0.3 and 341 m/min, respectively. Based on these values, the priors for n and C were selected to be normal distributions with mean values of 0.3 and 250 m/min, respectively, and standard deviations as 0.03 and 50 m/min, respectively. See Eq. (4):

$$n = \acute{N}(0.3,\ 0.03)\ \text{and}\ C = \acute{N}(250,\ 50)\ \text{m/min}, \qquad (4)$$

where $\acute{N}$ denotes a normal distribution and the values in the parentheses identify the mean and standard deviation, respectively. Since both n and C can be updated using experimental tool life values, the prior was selected to be a joint normal distribution as shown in Fig. 1. Also, n and C were assumed to be independent of each other and their joint probability density function (pdf) was determined by the multiplication of the two individual distributions determined from the literature review.

A Monte Carlo simulation was performed using the prior joint distribution of n and C to determine the prior distribution of tool
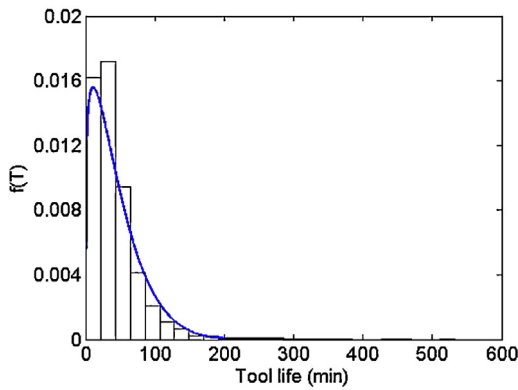
Fig. 2. Histogram showing the prior pdf of tool life at 1500 rpm. The best-fit beta distribution is also shown.

Fig. 4. Prior cdf of tool life from the Monte Carlo simulation. It is observed that the probability (see gray scale color bar) of obtaining a tool life of 50 min, for example, decreases with increasing spindle speed.

life, T. For the simulation, 1×10^5 random n and C samples were drawn from the prior joint distribution shown in Fig. 1 and the Taylor tool life curve was calculated for each pair using Eq. (1). The cutting speed was calculated using the relation $V = \pi DN$, where D is the tool diameter of 19.05 mm and N is the spindle speed (a range of 1500–7500 rpm was selected). As noted, no correlation was assumed between n and C. For the Monte Carlo simulation, the number of samples (1×10^5) was found to be large enough to ensure convergence to the true mean and standard deviation of the tool life distribution at any spindle speed. Fig. 2 shows the histogram of tool life values at 1500 rpm and a beta distribution fit to the histogram, where the histogram is normalized to obtain a unit area under the curve. As shown in Fig. 2, the beta distribution provides a good approximation of the actual histogram of tool life. Note that the single-sided distribution of the pdf is obtained due to the power law form of the Taylor tool life equation. The beta distribution in Fig. 2 represents the prior pdf of tool life at 1500 rpm. Similarly, beta distributions were fit to the histograms of tool life at other spindle speeds within the selected range; this formed the tool life prior distribution. Fig. 3 shows the prior cumulative distribution function (cdf) of T at $N = 1500$ rpm. The cdf was determined from the tool life histograms at this spindle speed. As shown in Fig. 3, the probability of obtaining a tool life greater than 300 min at 1500 rpm is effectively zero, while the probability of obtaining a tool life greater than 50 min (and less than 300 min) is 0.4. The uncertainty in the tool life, represented by the prior pdf and cdf of tool life, is due to the uncertainty in the values of n and C. This mimics the experimental reality that wear testing performed on two nominally identical tools will almost certainly yield slightly
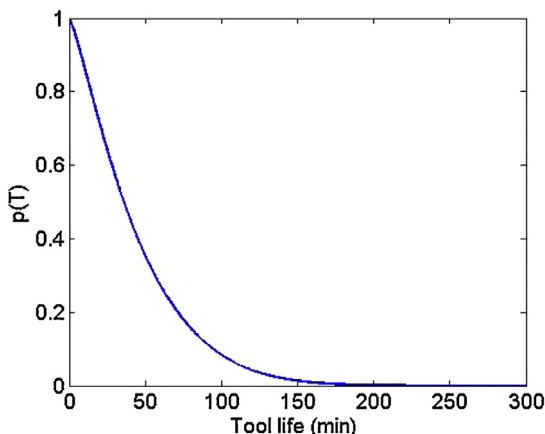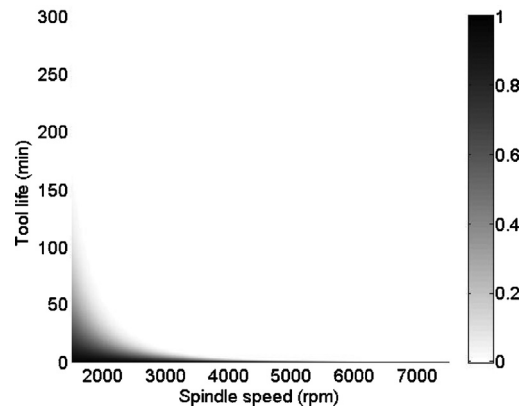
different n and C values. Fig. 4 shows the cdf of tool life, T, over the specified range of spindle speeds.

2.2. Experimental setup and results

The prior n and C joint distribution was updated using experimental tool life data to obtain the posterior joint distribution. In this section, the experimental steps followed to collect tool life data are described. Tool wear tests were completed using a 19.05 mm diameter single-insert Kennametal endmill (KICR073SD30333C) in down-milling. The insert was a 9.53 mm square uncoated carbide Kennametal insert (107888126 C9 JC) with zero rake and helix angles and a 15° relief angle. As noted, the workpiece material was 1018 steel. An atomic force microscope (AFM) was used to measure the topography of the carbide inserts prior to testing. Fig. 5 shows an example 50 μm × 50 μm measurement (256 line scans, no digital filtering) of the rake face. It is seen that there is a small chamfer with a 167° angle at the cutting edge. The roughness average for the rake face was 310 nm.

Wear tests were performed at a spindle speed of 1500 rpm ($V = 89.8$ m/min). The feed per tooth was 0.06 mm/tooth and the axial and radial depths of cut were 3 mm and 4.7 mm (25% radial immersion), respectively. The insert wear status was measured at regular intervals. To avoid removing the insert/tool from the spindle, a handheld microscope (60× magnification) was used to record digital images of the rake and flank surfaces. Fig. 6 shows the

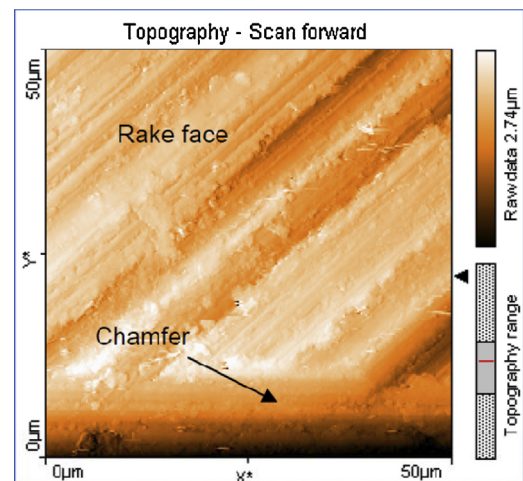**Fig. 3.** Prior cdf of tool life at 1500 rpm.

Fig. 5. AFM measurement of the carbide insert rake face and cutting edge chamfer.
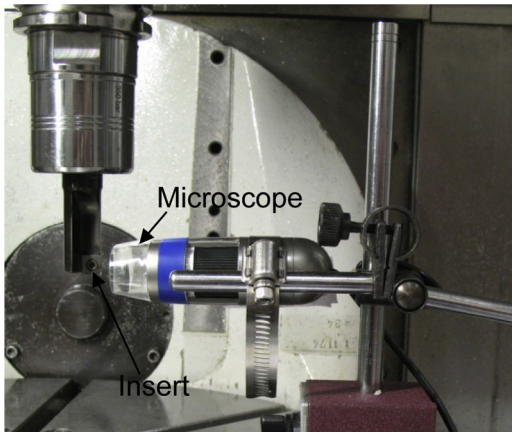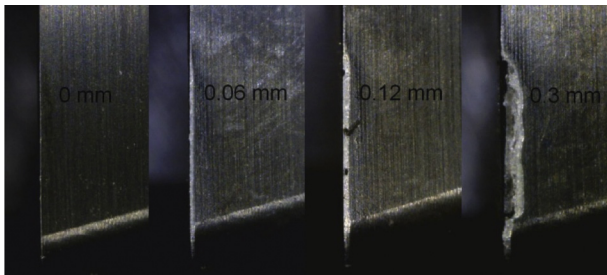
Fig. 6. Setup for in-process FWW measurement.

Fig. 7. Images of FWW at 60× magnification. The cutting times from left to right are {0, 78.5, 166.4, and 255.3} min.

microscope setup for recording the flank surface and measuring the FWW without removing the insert. The calibrated digital images were then used to identify the FWW (no crater wear was observed in these tests). Tool life, T, was defined as the time required for the insert to reach a FWW of 0.3 mm. Microscopic images of the relief face for selected cutting times are displayed in Fig. 7.

The time to reach a FWW of 0.3 mm was equal to 255.3 min for testing at 1500 rpm. Additional tests were performed at 3750 rpm and 6250 rpm. Table 1 shows the experimental results used for updating. The results of growth in FWW are displayed in Fig. 8. The 'o' symbols denote the intervals at which FWW was recorded. The tool life was linearly interpolated between adjacent measurement points if it exceeded 0.3 mm at the final measurement interval. As expected, the tool life reduced with increasing spindle speed. These experimental results were then used to update the prior joint distribution of n and C, and hence, the tool life, T, over a range of spindle speeds.

2.3. Bayesian updating using the discrete grid method

This section describes the discrete grid method used to determine the likelihood function and, subsequently, the posterior joint distribution [6]. The likelihood function was defined as the likelihood of observing experimental results for tool life, T, given n and C. The value of tool life, T, for the first experiment was 255.3 min at N = 1500 rpm (V = 89.8 m/min). The range of values of n and C

Table 1
Experimental tool life results used for updating.

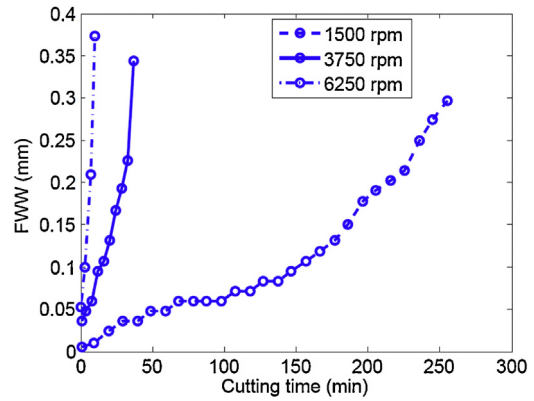| Test #	Spindle speed (rpm)	Cutting speed (m/min)	Tool life (min)
1	1500	89.8	255.3
2	3750	224.4	35.5
3	6250	374.0	8.5

Fig. 8. Variation of FWW with cutting time at various spindle speeds.

was divided into a 500 × 500 grid of points. The value of tool life, T, was calculated at each grid point (i.e., the selected {n, C} pair) using Eq. (1) at the value of V corresponding to the first experiment (V = 89.8 m/min). The distribution of tool life, T, at each grid point was assumed to be normal with an uncertainty (one standard deviation) equal to 10% of the experimental value. The T value calculated at each grid point was taken as the mean value. The likelihood at each grid point was taken to be the value of the pdf of T at the experimental value. To illustrate, consider n = 0.3 and C = 500 m/min. The tool life at the selected {n, C} combination at 1500 rpm using Eq. (1) is 305.9 min. The distribution was then determined at the selected {n, C} pair using the calculated tool life value (305.9 min) at that point as the mean and the assumed experimental uncertainty (one standard deviation equal to 10% of the experimental value). The likelihood of the {n, C} pair is the value of the pdf at the experimental value, which is equal to 0.0025 as shown in Fig. 9. In Fig. 9, 'o' represents the calculated tool life value at {n = 0.3, C = 500 m/min} and 'x' represents the experimental tool life obtained. The procedure was then repeated for all the grid points. The purpose of this approach was to evaluate the probability density that the selected {n, C} pair would yield the experimental value of tool life, T, using the deterministic model in the presence of uncertainty. This approach assigns all experimental non-repeatability to variations in n and C, i.e., it is assumed that there is no measurement uncertainty. Fig. 10 shows the likelihood for the first experimental result. Note that the color bands indicate the value of the pdf. The posterior was calculated using a point-by-point multiplication of the prior (Fig. 1) and likelihood (Fig. 10) and normalizing the product to obtain a unit volume under the pdf. Fig. 11 shows the posterior joint distribution of n and C after the first update.
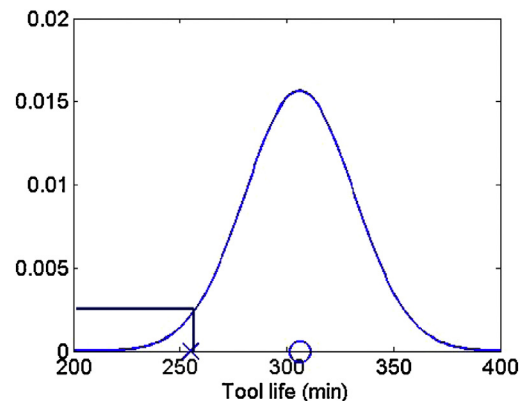

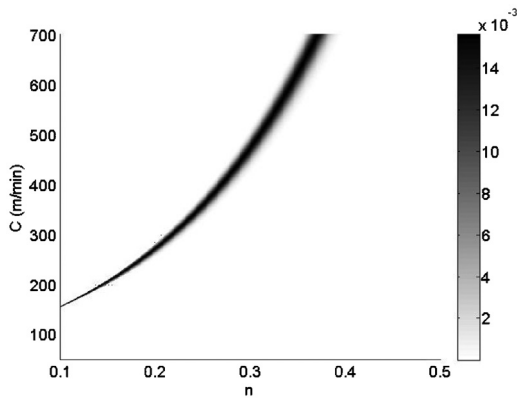Fig. 9. Likelihood calculation for n = 0.3 and C = 500 m/min for the experimental result of T = 255.3 min at 1500 rpm.

Fig. 10. Likelihood function for the first experimental result. The dark band identifies the {*n*, *C*} pairs which are more likely to yield the experimental result.
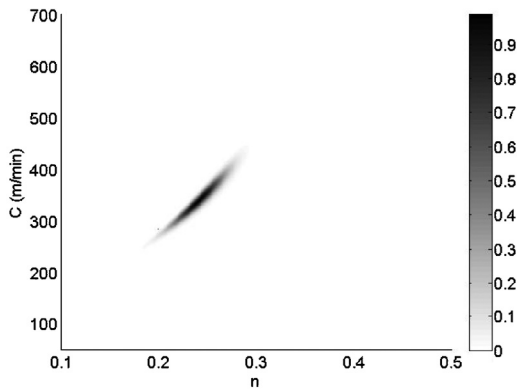

Fig. 11. Posterior joint distribution of *n* and *C* after the first update. Relative to Fig. 1 (the prior based on the literature review), it is seen that the new beliefs about *n* and *C* are quite different.

The same procedure was used to update the joint distribution of *n* and *C* for the remaining two experimental results. For each subsequent update, the posterior distribution from the previous update was the prior for the next update.

This process was repeated for each test. Fig. 12 shows the posterior joint distribution of *n* and *C* after three updates. The posterior mean and standard deviation for *n* are 0.33 and 0.011, respectively, and for *C* are 600.1 m/min and 35 m/min, respectively. Note that standard deviation has decreased with testing and updating, which indicates an improvement in belief (i.e., a reduction in uncertainty).

To determine the posterior joint distribution of tool life a Monte Carlo simulation was again performed. The posterior distribution
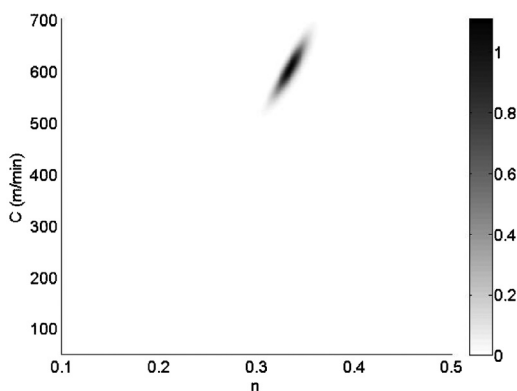

Fig. 12. Posterior joint distribution of *n* and *C* after three updates.
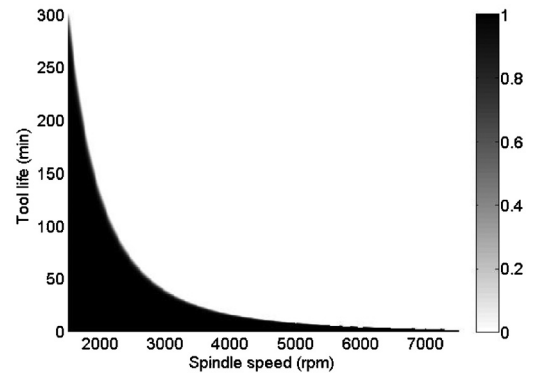
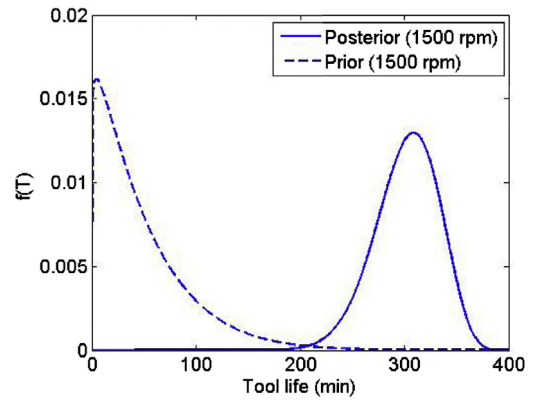

Fig. 13. Posterior cdf of tool life.

Fig. 14. Posterior and prior pdfs of tool life at 1500 rpm.

was approximated as a bivariate normal distribution. Random samples (1×10^5) were drawn from the bivariate normal posterior distribution of *n* and *C*; and tool life was then calculated at each sample point. Fig. 13 shows the tool life cdf over the specified range of spindle speeds. Figs. 14 and 15 compare the prior and posterior tool life pdf and cdf, respectively, at 1500 rpm. As shown in Fig. 15, the probability of observing a tool life of more than 300 min is now 0.4 as compared to zero according to the prior distribution (Fig. 3). Also, in comparison to the prior distribution of tool life (Fig. 4), there is a significant reduction in the posterior distribution spread.
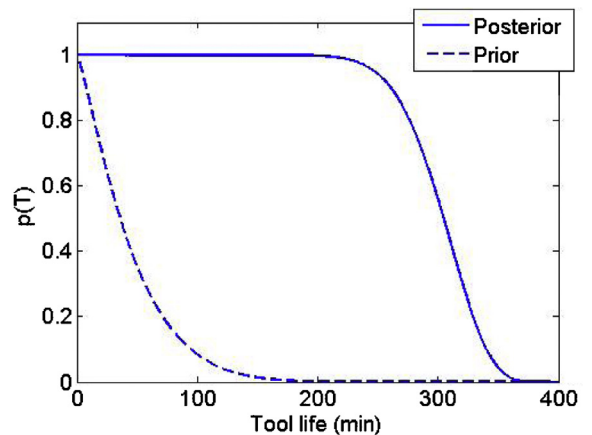

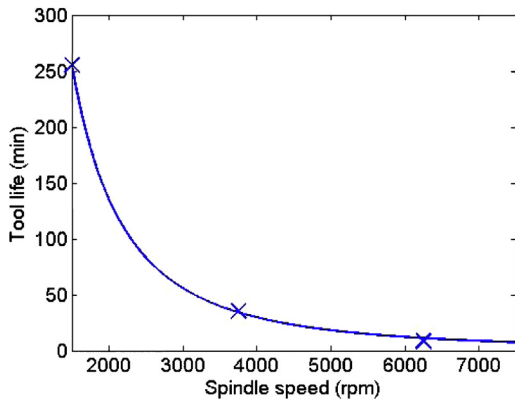**Fig. 15.** Posterior and prior cdfs of tool life at 1500 rpm.

Fig. 16. Least squares curve fit to the experimental data.

3. Tool life predictions

The posterior or the updated distribution of tool life can be used to predict tool life at spindle speeds other than the ones at which the tool wear experiments were performed. The posterior distribution was used to predict tool life at {2500, 5000, and 7500} rpm. Three tests were performed at each spindle speed to identify the non-repeatability. The tests were performed at the same parameters (other than spindle speed) as stated previously and the same procedure was followed to measure tool life. As before, tool life was set to be the time to reach a FWW of 0.3 mm. The tool life observed in these tests can be compared to the predicted posterior distributions of T at the corresponding spindle speeds (see Figs. 17–19). As seen from the figures, the predicted posterior distributions provide reasonable agreement with the experimental results.

4. Comparison with least squares curve fit

This section compares the tool life predictions using Bayesian inference to the least squares method. Fig. 16 shows the least squares curve fit and the experimental data ($R^2 = 0.9998$). A Monte Carlo simulation was performed to identify the fit uncertainty. The uncertainty (standard deviation) in the experimental result was again taken to be 10% of the measured value. 1×10^4 random samples were drawn from the normal joint distribution with the mean equal to the experimental value (from Table 1) and standard deviation equal to 10% of the experimental value. A least squares Taylor type curve was fit to each sample combination of tool life values. The mean and standard deviation from the Monte Carlo simulation
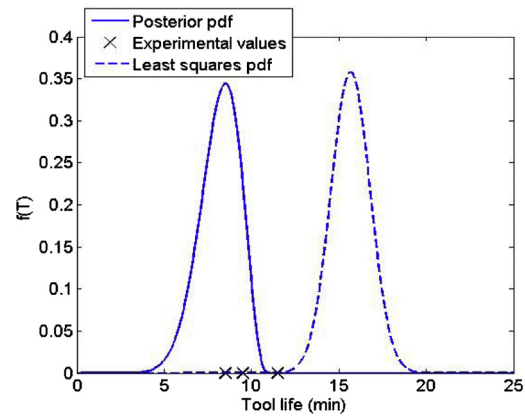

Fig. 17. Comparison of the posterior tool life pdf, the least squares curve fit prediction, and test results ('x') at 2500 rpm.

Fig. 18. Comparison of the posterior tool life pdf, the least squares curve fit prediction, and test results ('x') at 5000 rpm.

are 0.425 and 0.181 for n and 965.8 m/min and 65.8 m/min for C, respectively. The joint distribution of n and C was used to predict the distribution of tool life at 2500 rpm, 5000 rpm, and 7500 rpm. Table 2 shows a comparison of the values predicted by the least squares best fit and the Bayesian model and the experimental values. Because a statistical curve fit requires a large amount of data to achieve confidence in the fit parameters, it can be an expensive option for tool life testing. Fig. 17 shows the experimental values at 2500 rpm (denoted by 'x'), the posterior joint distribution after Bayesian updating, and the distribution of tool life predicted from the least squares fit. Figs. 18 and 19 show the same results for 5000 rpm and 7500 rpm, respectively. These figures show that the least squares prediction overestimates tool life. As noted, the least squares method requires a large amount of data to achieve confidence in the fit parameters. On the other hand, Bayesian inference takes into account prior beliefs and experimental evidence and, therefore, gives good results even for a small number of data points.

5. Effect of prior selection and likelihood uncertainty on tool life predictions

In Bayesian inference, the posterior distribution is a product of the prior and likelihood functions. Clearly, the posterior tool life predictions depend on the selected prior distribution and likelihood uncertainty. In this section, the influence of prior distribution and the likelihood uncertainty on the posterior is evaluated. First, the influence of the prior distribution on the
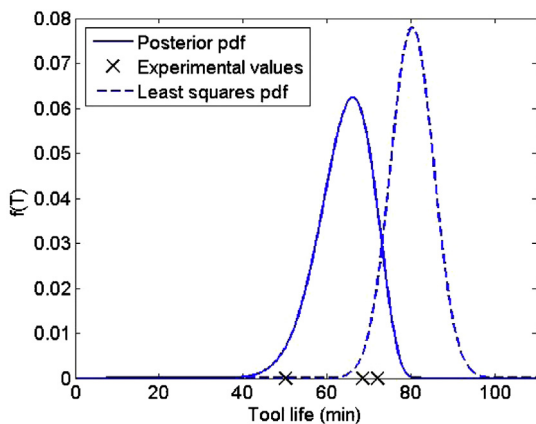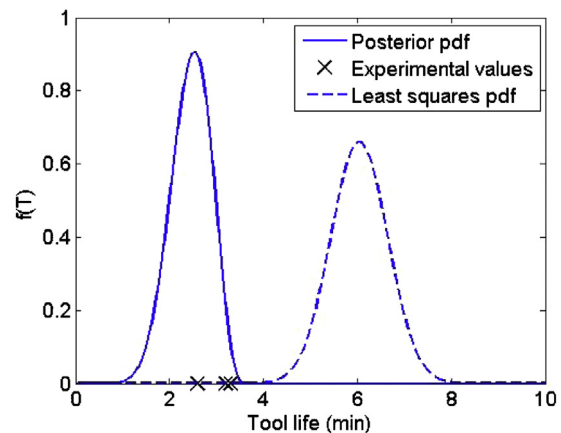

Fig. 19. Comparison of the posterior tool life pdf, the least squares curve fit prediction, and test results ('x') at 7500 rpm.

Table 2
Comparison between the experimental results and the values predicted from the least squares curve fit and Bayesian updating.

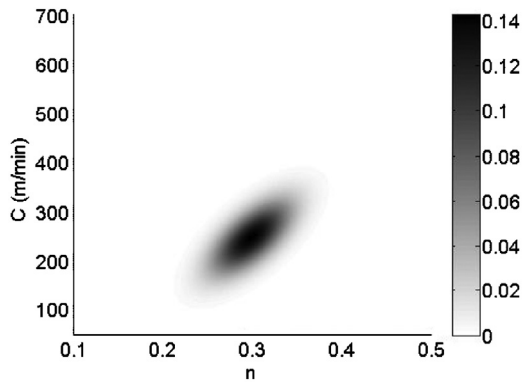| Test #	Spindle speed (rpm)	Cutting speed (m/min)	Tool life (min)	Curve fit prediction (min)	Bayesian prediction (min)
1	2500	149.6	50.1		
2	2500	149.6	68.5	(82.2, 5.2)	(62, 5.5)
3	2500	149.6	72.0		
4	5000	299.2	11.5		
5	5000	299.2	9.5	(15.7, 1.1)	(8, 0.7)
6	5000	299.2	8.5		
7	7500	448.8	2.6		
8	7500	448.8	3.3	(6.3, 0.6)	(2.4, 0.4)
9	7500	448.8	3.2		

Fig. 20. Prior joint distribution with 50% correlation between n and C. An elliptical shape is observed due to the correlation.
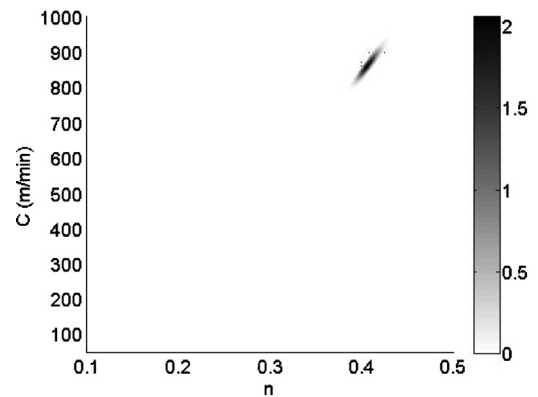

Fig. 22. Posterior joint distribution of n and C after three updates using the uniform prior.

posterior distribution was examined. In the prior (shown in Fig. 1), no correlation between n and C was assumed. However, correlation between the variables of interest may exist. Fig. 20 shows a prior with 50% correlation between n and C, i.e., selecting a particular n value partially identifies a corresponding C value. The prior updating was performed following the same procedure as described previously. The posterior mean and standard deviation for n were 0.33 and 0.012, respectively, and for C were 593.2 m/min and 34.8, respectively (see Fig. 21).

If no reported values or information about n and C are available (e.g., soon after a new alloy has been introduced), a non-informative prior can also be assumed. In this case, a uniform joint distribution over the range of values of n and C could be selected as the prior. A uniform joint distribution implies that the $\{n, C\}$ pair is equally likely to be anywhere within the domain. To illustrate this approach for the tool-material pair in this study, the minimum and

maximum values were taken as and 0.1 and 0.5 for n and 50 m/min and 1000 m/min for C. See Eq. (5):

$$n = U(0.1, 0.5) \text{ and } C = U(50, 1000), \tag{5}$$

where U represents a uniform distribution and the numbers in the parenthesis represent the minimum and maximum values, respectively. The updating procedure was repeated using these uniform prior distributions. The posterior mean and standard deviation for n were 0.408 and 0.017, respectively, and for C were 865 m/min and 56.2, respectively (see Fig. 22). Fig. 23 shows the tool life posterior pdfs using all three prior pdfs: normal with no correlation, normal with a 50% correlation, and uniform along with the experimental values ('x') and least squares prediction ('o'). The tool life posterior distribution has a lower mean for the correlated case, but the results do not differ substantially.
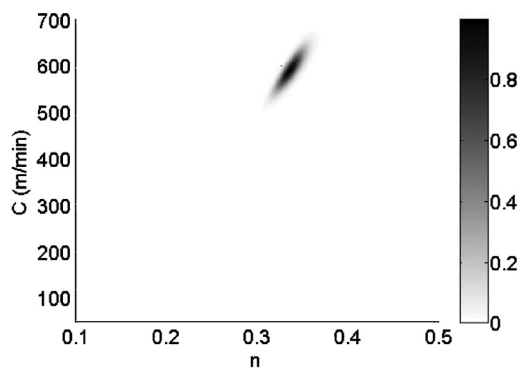

Fig. 21. Posterior joint distribution of n and C after three updates using a normal prior with 50% correlation between n and C.
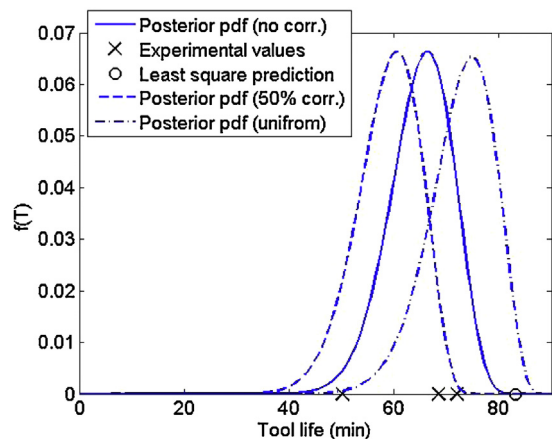

Fig. 23. Comparison of tool life predictions at 2500 rpm using different priors.

Table 3
Influence of prior pdf on posterior n and C mean and standard deviations.

	Prior	Mean n	Std. dev. n	Mean C (m/min)	Std. dev. C (m/min)
1	Uniform	0.408	0.0165	864.85	56.19
2	Normal (no corr.)	0.330	0.010	593.2	34.8
3	Normal (50% corr.)	0.335	0.0095	607.2	28.74

Table 4
Taylor tool life equation values for various uncertainty levels.

	% Std. dev.	Mean n	Std. dev. n	Mean C (m/min)	Std. dev. C (m/min)
1	1	0.416	0.002	935.64	8.46
2	5	0.384	0.008	785.3	29.9
3	10	0.330	0.010	593.2	34.8
4	20	0.278	0.018	414.3	41.8

The posterior n and C mean and standard deviation values are summarized in Table 3. The uniform prior pdf performs poorly in predicting tool life distribution as compared to the normal prior pdfs because it is less informative. In general, the prior should be selected to be as informative as possible, while accurately representing the user's beliefs.

The influence of the likelihood uncertainty on the posterior tool life distribution was also evaluated. In the discrete grid method used to determine the likelihood function, an uncertainty of 10% (one standard deviation) on the experimental tool life was assumed. To establish the sensitivity to this value, the uncertainty was varied (1%, 5%, 10% and 20%) and updating was performed for each case. Table 4 provides the mean and standard deviation of the posterior joint distribution of n and C for all four cases. Fig. 24 shows the 1500 rpm test likelihood functions for 5% and 20% standard deviations. Fig. 25 shows the posterior tool life pdf at 2500 rpm for different likelihood uncertainty percentages. The experimental values are denoted as 'x' and the least squares mean prediction as 'o'. Clearly, the assumed standard deviation affects
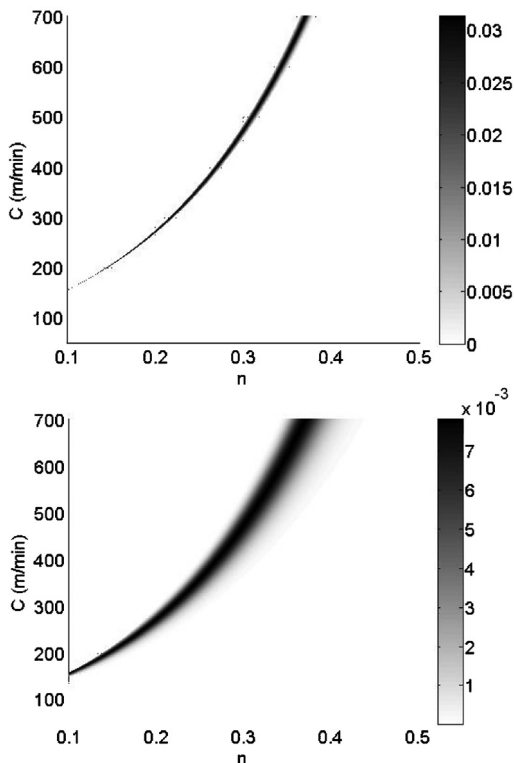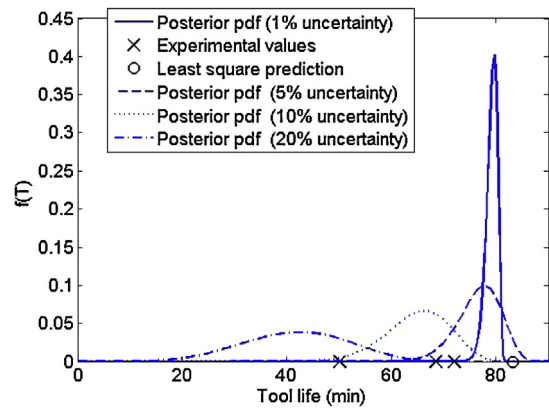

Fig. 25. Tool life predictions at 2500 rpm for different levels of likelihood uncertainty.

the spread of the likelihood function. As shown in Fig. 25, the posterior pdf obtained using a 20% likelihood uncertainty has a higher standard deviation than the posterior pdf obtained using 10% likelihood uncertainty. For any given experimental result, decreasing the experimental uncertainty reduces the spread of the likelihood function and, therefore, the posterior distribution. The spread of the posterior distribution determines the percentile value for the tool life prediction. For example, there is a 0.95 probability that the tool life would be more than the 95 percentile value. This value increases for smaller experimental uncertainty which makes the result less conservative (see Fig. 25). Thus, the prediction using a 20% uncertainty would be conservative, i.e., the tool life predicted would tend to be less than the experimental tool life. On the other hand, for low standard deviation values, the likelihood function approaches the deterministic result. This yields a less conservative tool life prediction in this case. As the likelihood uncertainty approaches zero, the posterior pdf standard deviation also approaches zero and the mean approaches the least squares prediction. In general, the value should be based on the uncertainty expected in the experimental value. Multiple experiments could be performed at a single spindle speed to determine this distribution, for example. From the values of tool life obtained in the prediction set, a 10% standard deviation is reasonable.

6. Conclusions

A Bayesian inference approach to tool life prediction was demonstrated. The Taylor tool life constants, n and C, were updated using milling tool life experimental results for a carbide insert-1018 steel combination. The prior joint distribution of n and C was determined by a literature review. The likelihood function was identified

Fig. 24. Likelihood functions for the 1500 rpm test with a 5% standard deviation (top) and a 20% standard deviation (bottom).

using the discrete grid method, where the likelihood gives the probability that a selected n and C combination would yield the experimental tool life value (using the deterministic model) in the presence of uncertainty. The posterior distribution was calculated as the normalized product of the prior and likelihood functions. The updated distribution was used to predict the values of tool life at different spindle speeds. The predicted distributions agreed with the experimental values of tool life.

Bayesian inference assigns a probability distribution over a range of the variable(s) of interest. The probability distributions of the predictions can be updated when new information is available (in the form of experimental results, for example). When this new information is obtained, uncertainty in the prior distributions can be reduced. Bayesian inference provides a way to combine prior data with experimental values to update beliefs about an uncertain variable. When combined with rational decision making theories, an optimal sequence of experiments and value gained from experimental results can also be determined.

Acknowledgements

References

[1] Taylor FW. On the art of cutting metals. Transactions of the ASME 1906;28:31–248.
[2] Vagnorius Z, Rausand M, Sorby K. Determining optimal replacement time for metal cutting tools. European Journal of Operational Research 2010;206:407–16.
[3] Wiklund H. Bayesian and regression approaches to on-line prediction of residual tool life. Quality and Reliability Engineering International 1998;14:303–9.
[4] Liu H, Makis V. Cutting-tool reliability assessment in variable machining conditions. IEEE Transactions on Reliability 1996;45(4):573–81.
[5] Astakhov VP. Tribology of metal cutting. Boston, MA: Elsevier; 2006.
[6] Gelman A, Carlin J, Stern H, Rubin D. Bayesian data analysis. Boca Raton, FL: Chapman and Hall/CRC Press; 2009.
[7] Tlusty J. Manufacturing process and equipment. Upper saddle River, NJ: Prentice Hall; 2000. p. 463.
[8] Stephenson DA, Agapiou JS. Metal cutting theory and practice. Boca Raton, FL: CRC, Press; 2006. p. 523.
[9] Kronenberg M. Machining science and application. Oxford: Pergamon Press; 1966. p. 170–204.
[10] Creese R. Introduction to manufacturing processes and equipment. New York, NY: Marcel Dekker, Inc.; 1999. p. 289.
[11] Cui Y, Fussell B, Jerard R, Esterling D. Tool wear monitoring for milling by tracking cutting force model coefficients. Transactions of the NAMRI/SME 2009;37:613–20.