



## Technical paper

## Spindle speed selection for tool life testing using Bayesian inference

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## ABSTRACT

According to the Taylor tool life equation, tool life is dependent on cutting speed (or spindle speed for a selected tool diameter in milling) and their relationship is quantified empirically using a power law exponent,  $n$ , and a constant,  $C$ , which are tool-workpiece dependent. However, the Taylor tool life model is deterministic and does not incorporate the inherent uncertainty in tool life. In this work, Bayesian inference is applied to estimate tool life. With this approach, tool life is described using a probability distribution at each spindle speed. Random sample tool life curves are then generated and the probability that a selected curve represents the true tool life curve is updated using experimental results. Tool wear tests are performed using an inserted (uncoated) carbide endmill to machine AISI 1018 steel. The test point selection is based on the maximum value of information approach. The updated beliefs are then used to predict tool life using a probability distribution function.

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## 1. Introduction

Tool wear can present a significant limitation to machining productivity. Taylor [1] first defined an empirical relationship between tool life and cutting speed, referred to as the Taylor tool life equation:

$$vT^n = C, \quad (1)$$

where  $v$  is the cutting speed in m/min,  $T$  is the tool life in minutes, and  $n$  and  $C$  are constants which depend on the tool-workpiece combination. The constant  $C$  is defined as the cutting speed required to obtain a tool life of 1 min. Tool life can be defined as the time required to reach a predetermined flank wear width, FWW [2–4], although other wear criteria such as dimension tool life, dimension wear rate (defined as the rate of shortening of the cutting tip in the direction perpendicular to the machined surface taken within the normal wear period) [5], crater wear [6–8], and volumetric wear [9–11], may also be applied. The Taylor tool life model is deterministic in nature, but uncertainty always exists in practice due to: (1) factors that are unknown or not included in the model (because of the complex nature of the tool wear process); and (2) tool-to-tool performance variation. For these reasons, tool wear is often considered to be a stochastic process and, therefore, difficult to predict [12]. Different methods have been demonstrated in the literature

for tool life predictions and estimation such as design of experiments [13,14], neural networks [15–18], fuzzy systems [19,20], and Bayesian inference [12].

Bayesian inference, which forms a normative and rational method for belief updating, is applied in this paper. Let the prior distribution about an uncertain event,  $A$ , at a state of information,  $\&$ , be  $\{A|\&\}$ , the likelihood of obtaining an experimental result  $B$  given that event  $A$  occurred be  $\{B|A, \&\}$ , and the probability of receiving an experimental result  $B$  (without knowing  $A$  has occurred) be  $\{B|\&\}$ . Bayes' rule is used to determine the posterior belief about event  $A$  after observing the experiment results,  $\{A|B, \&\}$ , as shown in Eq. (2). Using Bayes' rule, information gained from experiments can be combined with the prior prediction to obtain a posterior distribution.

$$\{A|B, \&\} = \frac{\{A|\&\}\{B|A, \&\}}{\{B|\&\}} \quad (2)$$

The Taylor tool life model assigns a deterministic value to tool life for a selected cutting speed. In contrast, Bayesian inference assigns a probability distribution to the tool life value at a particular cutting speed. The prior, or initial user belief about tool life, can be based on theoretical considerations, expert opinions, past experiences, or data reported in the literature; the prior should be as chosen to be as informative as possible. The prior is represented as a probability distribution and, using Bayes' theorem, this probability distribution can be updated when new information is available (from tool wear experiments, for example). As a result,

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Bayesian inference enables a model to both incorporate uncertainty and “learn” (update beliefs based on experimental results).

## 2. Bayesian inference of the Taylor tool life model

Bayesian inference provides a rigorous mathematical framework of belief updating about an unknown variable when new information becomes available. In the Taylor tool life model (Eq. (1)), there is uncertainty in the exponent,  $n$ , and the constant,  $C$ . As a result, there is uncertainty in the tool life,  $T$ . The prediction of the Taylor tool life curve can be simulated by generating  $N$  sample tool life curves, or sample paths, each representing the true tool life curve with equal probability. Therefore, the prior probability that any sample path is the true Taylor tool life curve is  $1/N$ . The sample paths generated in this way may be used as the prior for Bayesian inference. The prior can be updated by applying Bayes' rule to experimental test results. For each sample path, Bayes' rule can be written as the following product.

$$p(\text{path} = \text{true tool life curve} | \text{test result}) \\ = \frac{p(\text{test result} | \text{path} = \text{true tool life curves})}{p(\text{test result}) \cdot p(\text{path} = \text{true tool life curve})}$$

Here,  $p(\text{path} = \text{true tool life curve})$  is the prior probability that a given path is the true tool life curve. The probability is assumed to be  $1/N$  before any testing is completed. Also,  $p(\text{test result} | \text{path} = \text{true tool life curve})$  is referred to as the likelihood,  $p(\text{test result})$  is a normalization constant, and  $p(\text{path} = \text{true tool life curve} | \text{test result})$  is the posterior probability of the sample tool life curve given a test result. In this study, the prior sample paths were generated using random samples from an  $n/C$  joint probability density function (pdf). The initial (prior)  $n$  and  $C$  distributions were selected using a literature review. In general, the decision maker should try to use all available information to generate the sample paths. Bayes' rule was then used to update the probability that each sample path was the true tool life curve.

According to Bayes' rule, the posterior distribution is proportional to the (normalized) product of the prior and the likelihood. For multiple experimental results, the posterior after the first update becomes the prior for the second update and so on, where the posterior probabilities of each sample path must be normalized so that the sum of the probabilities for all paths is one. In a milling operation, other factors, such as feed rate and axial/radial depths of cut, may also affect tool life in addition to the cutting speed. However, since cutting speed is typically the strongest factor, Bayesian updating was performed using Eq. (1). Without loss of generality, the procedure can be extended to other forms of the Taylor tool life equation.

### 2.1. Establishing the prior

Tool wear experiments were performed using an uncoated carbide (inserted) tool to mill AISI 1018 steel. As noted, a literature review was completed to determine the prior distributions of the Taylor tool life constants,  $n$  and  $C$ . Using the corresponding joint pdf, random samples were used to generate the prior sample paths of the tool life curves. Stephenson and Agapiou [21] reported the value of  $n$  to be in the range of 0.2–0.25 for uncoated carbide tools and  $C$  to be around 100 m/min for rough finishing of low carbon steels. Kronenberg [22] reported values of  $n$  and  $C$  to be in the range of 0.3–0.5 and 160–200 m/min, respectively, for machining steel with a carbide tool. Creese [23] reported typical  $n$  and  $C$  values for machining medium carbon steel with a carbide tool to be 0.32 and 240 m/min, respectively. Cui et al. [24] performed wear experiments using a carbide insert and 1018 steel workpiece. Values of  $n$  and  $C$  were reported to be 0.3 and 341 m/min, respectively. In a separate study

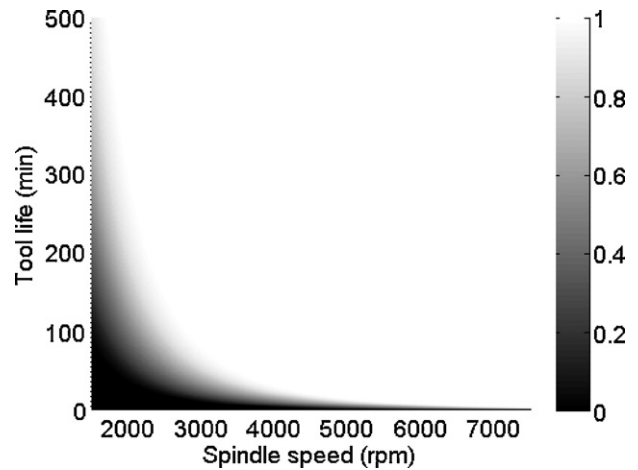


Fig. 1. Prior cdf of tool life. The gray-scale color bar represents the probability of tool failure at a selected tool life for any spindle speed in the domain.

conducted by the authors, the means of the constants  $n$  and  $C$  for the given tool-work piece combination were found to be 0.33 and 600 m/min.

Based on these values, the priors for  $n$  and  $C$  were selected to be uniform distributions with minimum values of 0.3 and 400, respectively, and maximum values of 0.35 and 700, respectively. A uniform distribution implies that it is equally likely for the true  $n$  and  $C$  value to be anywhere in the selected range. This is expressed using:

$$n = U(0.3, 0.35) \quad \text{and} \quad C = U(400, 700),$$

where  $U$  denotes a uniform distribution and the values in the parentheses identify the minimum and maximum values, respectively.

Note that relatively large prior distributions of  $n$  and  $C$  were chosen to improve the probability that the true tool life curve existed within the prior sample paths. The prior  $n$  and  $C$  distributions were taken as a joint pdf, where the two constants were independent of each other. Random samples were drawn from the prior joint pdf of  $n$  and  $C$  and the Taylor tool life curve was calculated for each  $\{n, C\}$  pair; this exercise was repeated  $1 \times 10^4$  times. The cutting speed was calculated using  $v = \pi d \Omega$ , where  $d$  is the tool diameter of 19.05 mm and  $\Omega$  is the spindle speed in rev/min (a range of 1500–7500 rpm was selected). The prior probability that any sample paths is the true tool life curve for this case is  $1 \times 10^{-4}$ . The collection of prior sample paths could then be used to determine the cumulative density function (cdf) of tool life at any spindle speed in the domain. Fig. 1 shows the prior cumulative distribution of tool life as a function of spindle speeds. The color bar gives the probability of tool failure at a selected tool life for any spindle speed in the domain. Naturally, the probability of failure decreases with spindle speed for a particular tool life value. Fig. 2 shows the histogram of tool life values at 5000 rpm. Fig. 3 shows the prior cdf at 5000 rpm that describes the probability of tool failure for a selected tool life. As expected, the probability of failure is small for low tool life values.

### 2.2. Likelihood function

Tool life is stochastic. If a tool life experiment is repeated under the same conditions, it is unlikely that exactly the same tool life would be obtained over multiple trials. The likelihood function is designed to account for this behavior. To illustrate, consider that a tool life of 7 min was obtained at 5000 rpm. The user might believe that a tool life between 5 min and 9 min is therefore very likely if the experiment was repeated. The user may also believe that it is

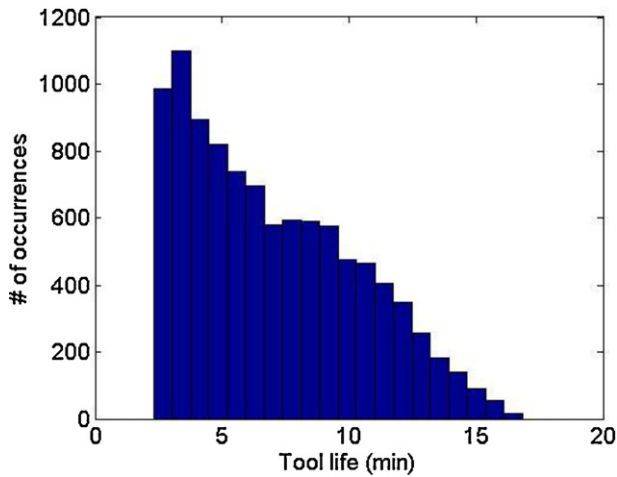


Fig. 2. Histogram of tool life at 5000 rpm from the prior.

not very likely that the tool will last less than 4 min or greater than 10 min based on the initial result. This information is taken into account using the likelihood function provided in Eq. (3):

$$l = e^{-(T-T_m)^2/k}, \tag{3}$$

where  $l$  is the likelihood function,  $T_m$  is the measured tool life,  $T$  is the tool life value for a sample curve at the experimental spindle speed, and  $k$  depends on the tool life distribution. Because the likelihood function is expressed as a non-normalized normal distribution,  $k = 2\sigma^2$ , where  $\sigma$  is the standard deviation of tool life. The likelihood function describes how likely it is that the sample tool life curve is the correct curve given the measurement result at a particular spindle speed. If the tool life curve value is near the measurement result, then the likelihood value is high. Otherwise, it is low. The likelihood function defined in Eq. (3) does not completely reject paths which differ significantly from the experimental result; it simply yields a small value. To illustrate, again consider a measured tool life of 7 min at 5000 rpm. At 5000 rpm, each sample tool life curve will have a value of tool life depending on the  $\{n, C\}$  pair used to generate the sample path. The likelihood function can be interpreted as assigning weights to sample paths from zero to unity, where zero indicates that the selected combination is not likely at all and unity identifies the most likely combination. The likelihood for each sample tool life curve was calculated using Eq. (3). The parameter  $T$  in the equation is the tool life at the experimental

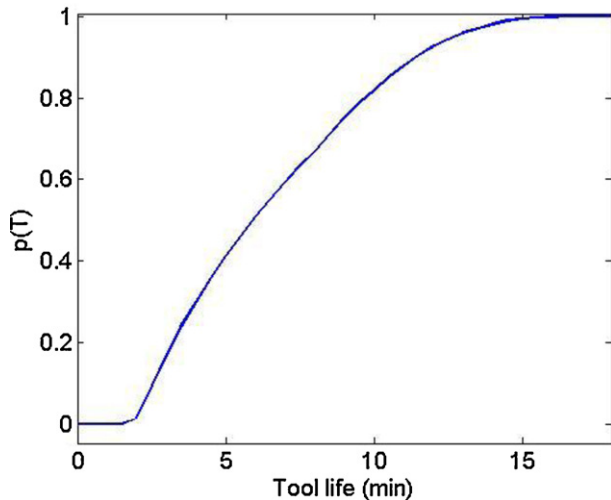


Fig. 3. Prior cdf of tool life at 5000 rpm.

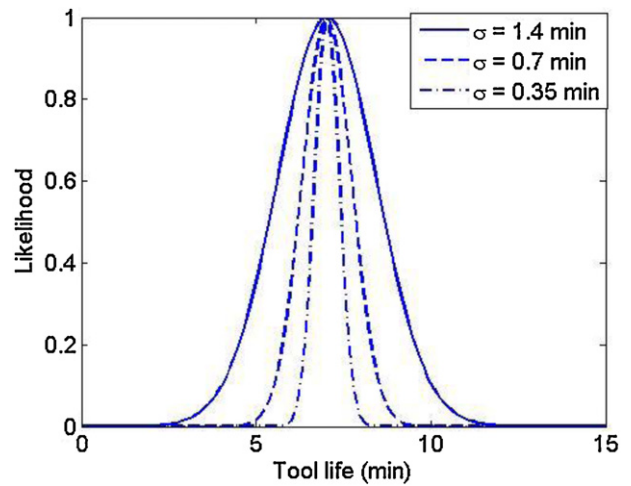


Fig. 4. Likelihood for various uncertainty levels based on a measured tool life of 7 min at 5000 rpm.

spindle speed (in this example, 5000 rpm) for a selected sample tool life curve.

The value of  $k$  is selected by the user based on his/her beliefs about the experimental uncertainty. For this study, the standard deviation for an experimental result was assumed to be 20% of the measured value. Fig. 4 shows the likelihood function for  $T_m = 7$  min at 5000 rpm for different  $\sigma$  values (and, therefore,  $k$  values). As seen in the figure, increased uncertainty (higher  $\sigma$ ) widens the likelihood function so that comparatively higher weights are assigned to sample curves far from the experimental result. Subsequently, larger uncertainty yields a more conservative estimate of tool life. Although the value of  $\sigma$  is considered constant in this study, it can also be expressed as a function of spindle speed.

### 2.3. Bayesian updating

As noted, the likelihood function describes how likely it is that the sample tool life curve is the correct curve given the measurement result at a particular spindle speed. The prior probability for each path is  $1/N$ , where  $N$  is the number of sample paths and the likelihood value is determined using Eq. (3). According to Bayes' rule, the posterior distribution is obtained by a multiplication of the prior and the likelihood. The posterior probability for each path is then normalized so that the sum is equal to unity. At each spindle speed, the updated probabilities of sample tool life curves provide an updated distribution of tool life. Fig. 5 displays updated posterior distribution given  $T_m = 7$  min at 5000 rpm. Fig. 6 shows a comparison between the prior and posterior tool life pdfs at 5000 rpm. For the posterior cdf calculation, the updated probabilities, or weights, of the sample paths must be considered.

### 3. Value of information

The combination of Bayesian inference and decision analysis enables a dollar value to be placed on the information gained from an experiment prior to performing it. The value of information,  $VOI$ , may be defined as the difference between the expected profit before testing and the expected profit after testing. For a fixed sales price,  $VOI$  is the expected cost prior to testing minus the expected cost after testing. In simple terms,  $VOI$  identifies the monetary gain from performing an experiment. Note that the value of information is the 'expected' value obtained after an experiment; it is actually calculated before performing the test. The primary motivation for calculating  $VOI$  is to design the experimental study. The

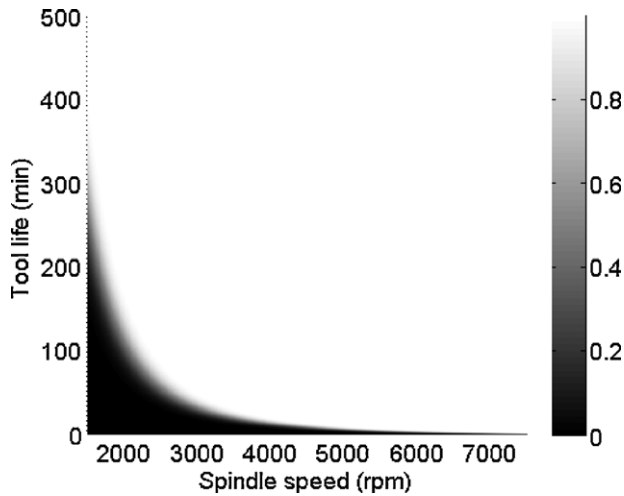


Fig. 5. Posterior cdf given an experimental tool life of 7 min at 5000 rpm.

experimental test point is chosen which adds the most (expected) value to the profit. In addition, if the expected cost of performing an experiment is greater than  $VOI$ , it is probably not a good idea to experiment at all.

### 3.1. Determining the cost

Before calculating the value of information, it is necessary to determine the cost of performing the test given the selected operating conditions. In this study, it was assumed that a pocket was to be machined. The volume of material to be removed,  $V$ , was assumed to be  $1 \times 10^5 \text{ mm}^3$ . The machining cost,  $C_m$ , can be written as shown by Tlusty [25]:

$$C_m = t_m r_m + \frac{(C_{te} + t_{ch} r_m) t_m}{T}, \quad (4)$$

where  $t_m$  is the machining time,  $r_m$  is the cost per unit time to operate the machine,  $C_{te}$  is the cost/tool edge for an inserted cutter or cost/tool for a solid endmill,  $t_{ch}$  is the tool changing time, and  $T$  is the tool life. The machining time is calculated from the material removal rate,  $MRR$ , and the volume to be removed. See Eqs. (5) and (6):

$$MRR = \Omega N_t a b f_t \quad (5)$$

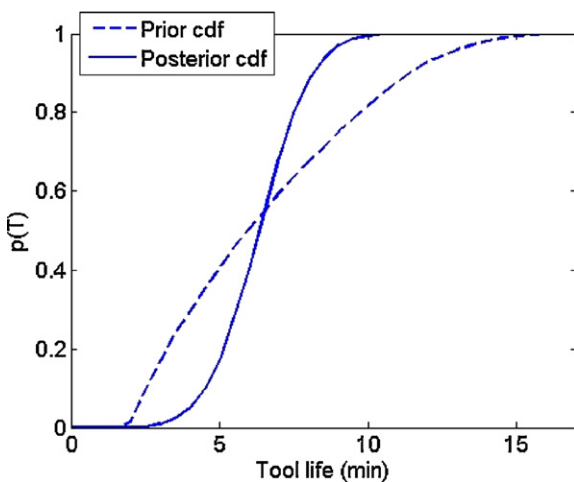


Fig. 6. Prior and posterior cdf of tool life at 5000 rpm. The vertical axis represents the probability of tool failure.

Table 1  
Cost variables and values.

Variable	Value	Unit
$d$	19.05	mm
$f_t$	0.06	mm/tooth
$a$	4.76	mm
$b$	3	mm
$N_t$	1	–
$r_m$	10	\$/min
$V$	$1 \times 10^5$	$\text{mm}^3$
$C_{te}$	10	\$
$t_{ch}$	2	min
$L$	1000	\$

$$t_m = \frac{V}{MRR}, \quad (6)$$

where  $N_t$  is the number of teeth,  $a$  is the radial depth of cut,  $b$  is the axial depth of cut, and  $f_t$  is the feed per tooth. Because tool life has a probability of failure associated with it (see Fig. 6), the machining cost is modified to be an expected cost. This requires that a user-defined cost of tool failure,  $L$ , be included in the cost expression, where tool failure denotes that the worn tool has exceeded the permissible wear limit (for example, maximum FWW) and a tool failure results in additional cost. This may be due to loss of machining time, reduced productivity, cost of reworking the part, or discarding the part altogether. The term  $L$  takes into account all the costs associated with a tool failure. The value of  $L$  need not be determined accurately;  $L$  can be interpreted as the value that must be paid to the user that will make him/her indifferent to a tool failure. The expected machining cost at any spindle speed,  $E(C_m)$ , is given by Eq. (7), where  $p$  is the probability of tool failure (obtained from the posterior cdf as shown in Fig. 5, for example). The best spindle speed for testing corresponds to the lowest expected cost from Eq. (7). To demonstrate the concept of expected cost, let the cost of machining at some spindle speed be \$10. Let the probability of tool failure be 0.1 and let  $L$  also be equal to \$10. For a production run of 100 parts, there will be (on average) ten tool failures because  $p=0.1$ . This will incur an additional cost of \$10 for each of the 10 tool failures. Thus, the expected total cost of machining 100 parts is  $\$100 \times 10 + \$10 \times 10 = \$1100$ . The expected machining cost of a single part is then \$11. The values of the variables used to calculate machining cost for this study are listed in Table 1.

$$\begin{aligned} E(C_m) &= (1 - p) \left( t_m r_m + \frac{(C_{te} + t_{ch} r_m) t_m}{T} \right) \\ &\quad + p \left( t_m r_m + \frac{(C_{te} + t_{ch} r_m) t_m}{T} + L \right) \\ &= \left( t_m r_m + \frac{(C_{te} + t_{ch} r_m) t_m}{T} \right) + pL \end{aligned} \quad (7)$$

To illustrate the approach, consider the prior and posterior tool life cdfs shown in Fig. 6. Each tool life at 5000 rpm has a probability of failure associated with it. The tool life values,  $T$ , and the corresponding probability of tool failure,  $p$ , were used to calculate the expected cost for each 5000 rpm tool life by Eq. (7). Fig. 7 shows the prior cost of machining at 5000 rpm as a function of tool life. At higher tool life values ( $T > 9$  min), the term  $pL$  dominates the cost equation. At small values of tool life ( $T < 3$ ), the value of  $p$  is close to zero. Also, at small tool life values, where a higher number of tool changes is required, the ratio  $(C_{te} + t_{ch} r_m) t_m / T$  dominates the cost equation. From the prior, the minimum machining cost for a spindle speed of 5000 rpm is \$526.70 at 2.51 min with a probability of failure equal to 0.015. Fig. 8 shows the posterior cost at 5000 rpm. The minimum machining cost is \$431.10 at 4.13 min with a probability of failure equal to 0.028. This procedure was repeated at all spindle speeds. Fig. 9 shows the minimum cost as a function of

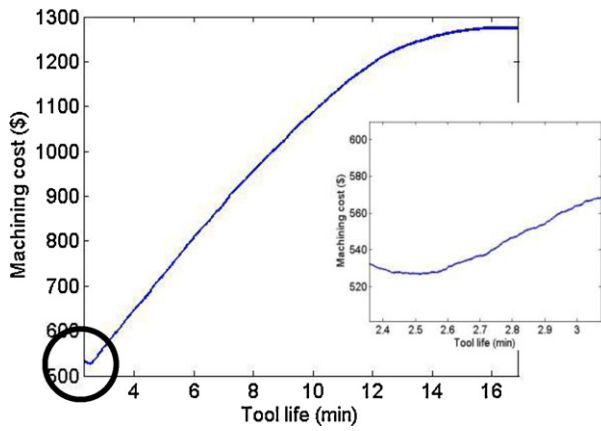


Fig. 7. Machining cost based on the prior at 5000 rpm as a function of tool life.

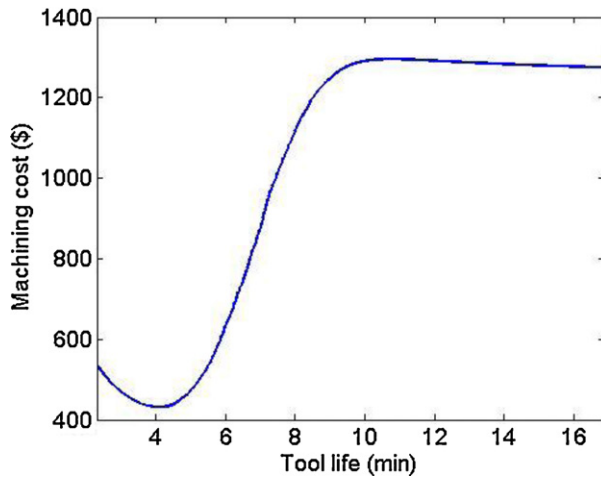


Fig. 8. Machining cost based on the posterior at 5000 rpm as a function of tool life.

spindle speed based on the prior and the posterior distributions of tool life. Table 2 summarizes the smallest value from the minimum cost curve and optimum tool life based on the prior and posterior. Note that the posterior distribution was determined by assuming a measured tool life of 7 min at 5000 rpm for this example.

Risk neutrality of the user was assumed in this study. However, for a risk averse user, an acceptable threshold value of  $p$  can be set.

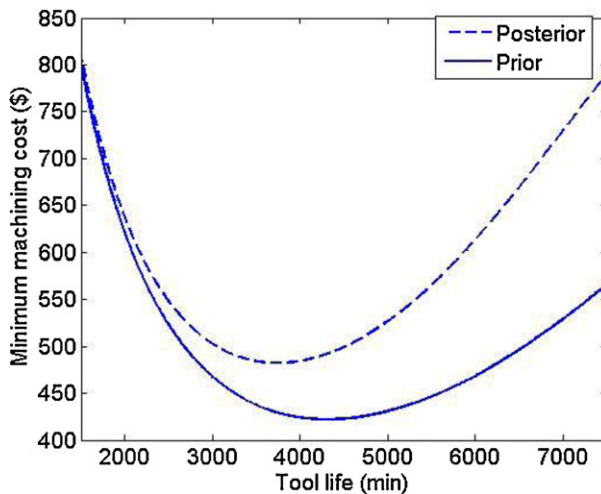


Fig. 9. Minimum machining cost based on prior and posterior tool life distributions.

Table 2  
Optimum machining cost and tool life for prior and posterior distributions.

	Machining cost (\$)	Tool life (min)	Spindle speed (rpm)	Probability of tool failure
Prior	482.6	5.75	3740	0.008
Posterior	422.2	6.31	4292	0.021

This means that a risk averse user can say that he/she is not willing to operate at any spindle speed where a threshold value of  $p$  is exceeded. In that case, the expected cost at a spindle speed would be calculated using tool life values with  $p$  values less than the user defined threshold. Risk aversion leads to a conservative estimate of optimum tool life and the corresponding spindle speed.

### 3.2. Selecting the optimum spindle speed

To determine an optimum test spindle speed, the test spindle speed range was divided into discrete intervals (50 rpm increments were used here). As a heuristic, it was assumed that a test at a selected spindle speed gives the expected tool life at that speed. (Recall that the value of information is determined before actually performing the test.) The uncertainty in the experimental tool life was assumed to be 20%. It was also assumed that 1000 pockets need to be machined. The value of information is then given by:

$$VOI = 1000 \cdot (E(\text{Cost before test}) - E(\text{Cost after test}) - E(\text{Cost of performing the test}))$$

where  $VOI$  provides the value of performing an experiment at a selected spindle speed. A negative value states that that cost of performing the experiment is more than the expected value to be gained, which suggests that no additional testing is necessary. The procedure to calculate  $VOI$  follows. First, the expected cost before testing is calculated from the prior distribution (see Fig. 1). Second, at each test spindle speed, the prior is updated using the expected tool life at that speed from the prior distribution. Third, the expected cost after testing is calculated using the updated posterior cdf. As shown in Eq. (7), the expected cost of performing a test,  $E(C_t)$ , is the sum of the expected cost for the experiment (the product of the expected tool life and rate of machining), the tool cost, and the expected material cost. The material cost is the product of the expected tool life, material removal rate, and the cost per unit volume of the material. The cost per unit volume of AISI 1018 steel was taken as  $6 \times 10^{-5}$  \$/mm<sup>3</sup>. Note that the material removal rate is dependent on spindle speed (the number of teeth, feed per tooth, and axial/radial depths were assumed to be fixed in this study).

$$E(C_t) = E(T)r_m + C_{te} + 6 \times 10^{-5}E(T)MRR \tag{8}$$

To illustrate, consider three test speeds, {1500, 5000 and 7500} rpm. If a test was to be performed at these speeds, it was assumed that the measured tool life would be equal to the expected tool life at those speeds. The expected tool life (determined from the prior) at {1500, 5000, 7500} rpm is {302, 7, 2} min, respectively. Table 2 shows the prior and posterior minimum cost given a measured tool life of 7 min at 5000 rpm. The same procedure is followed to calculate the posterior cost given an experimental result of 302 min at 1500 rpm and 2 min at 7500 rpm. The posterior minimum cost after testing was then used to calculate  $VOI$ . Note that it costs more to perform a test at 1500 rpm than at 7500 rpm because the tool generally lasts longer at lower cutting speeds. Table 3 summarizes the three results. It is observed that it is most profitable to complete a test at 1500 rpm. The procedure was repeated at all speeds from 1500 rpm to 7500 rpm with an interval of 50 rpm. Fig. 10 shows  $VOI$  as a function of spindle speed. It is seen that  $VOI$  is highest at 1900 rpm. Therefore, the best test speed is 1900 rpm.

**Table 3**  
Value of information.

Test speed (rpm)	Prior cost (\$)	$E(T)$ (min)	Posterior cost (\$)	Test cost (\$)	
1500	482.6	302.0	413.3	3056.8	82,410
5000		7.0	422.2	83.4	79,660
7500		2.0	421.4	31.2	78,470

#### 4. Experimental setup and results

In this section, the experimental procedure used to collect tool life data is described. Down-milling tool wear tests were completed using a 19.05 mm diameter single-insert Kennametal endmill (KICR073SD30333C). The insert was a 9.53 mm square uncoated carbide Kennametal insert (107888126 C9JC) with zero rake and helix angles and a  $15^\circ$  relief angle. The workpiece material was AISI 1018 steel. From the VOI analysis, the best test speed was determined to be 1900 rpm. The first wear test was performed at a spindle speed of 1900 rpm ( $v = 113.7$  m/min). The feed per tooth was 0.06 mm/tooth and the axial and radial depths of cut were 3 mm and 4.7 mm (25% radial immersion), respectively. The insert wear status was measured at regular intervals. To avoid removing the insert/tool from the spindle, a digital microscope (60 $\times$  magnification) was used to record the rake and flank surfaces; see Fig. 11. The calibrated digital images were used to identify the FWW (no crater wear was observed in these tests). Tool life was defined as the time required for the insert to reach a FWW of 0.3 mm. Fig. 12 shows the variation of FWW with cutting time for tests at 1900 rpm. The 'o' symbols denote the intervals at which FWW was recorded. Microscopic images of the relief face for selected cutting times are displayed in Fig. 13. The tool life was linearly interpolated between adjacent intervals if it exceeded 0.3 mm at the final measurement interval. The time to reach a FWW of 0.3 mm was equal to 164 min for testing at 1900 rpm. The prior distribution (see Fig. 1) was now updated using the tool life result of 164 min at 1900 rpm. A 20% uncertainty was assumed on the experimental tool life value ( $\sigma = 32.4$  min).

The procedure to update the probability of each sample path is described next. First, the likelihood value for each sample path was calculated using Eq. (3) assuming  $\sigma = 32.4$  min. Next, the posterior probability of each sample tool life curve was calculated from the product of the prior probability ( $1/N$ ) and the likelihood; these results were then normalized to obtain a sum equal to unity. Fig. 14

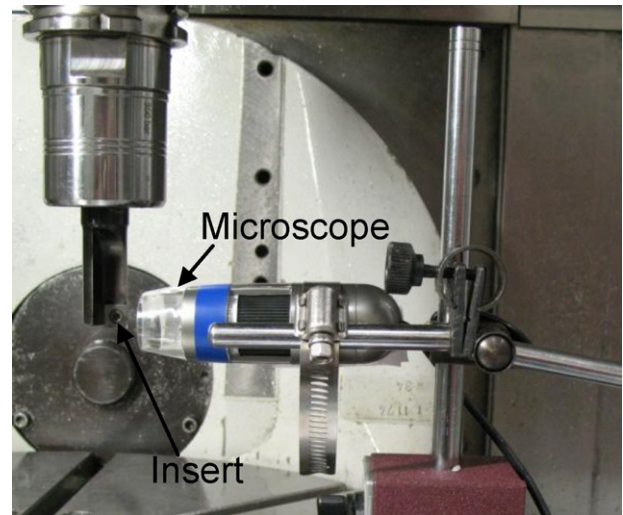


Fig. 11. Setup for in-process FWW measurement.

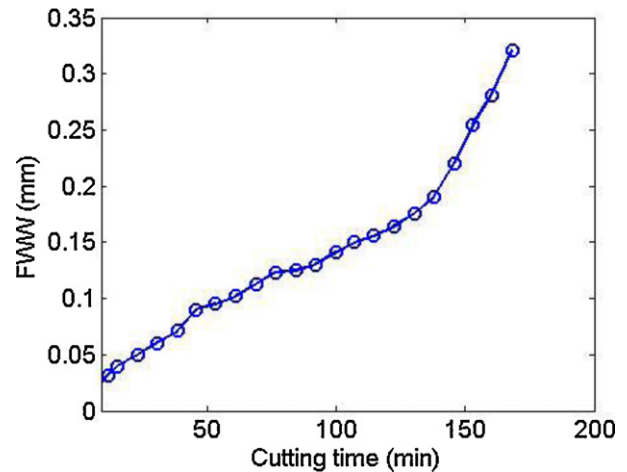


Fig. 12. Increase in FWW with cutting time at 1900 rpm.

shows the updated posterior distribution using the measured result of 164 min at 1900 rpm. Fig. 15 shows the prior and posterior distribution of tool life at 1900 rpm. In the figure, the prior probability of obtaining a tool life less than 100 min was 0.35 and 0.65 for a tool life less than 164 min. For the posterior distribution, the probability of obtaining a tool life less than 100 min is 0.02 and 0.50 for a tool life less than 164 min.

The VOI calculation procedure was repeated to find the next test speed. After the first update, the posterior was used as the new prior. Again, before performing the tests, it was assumed that a test at any speed would yield the expected tool life at that speed.

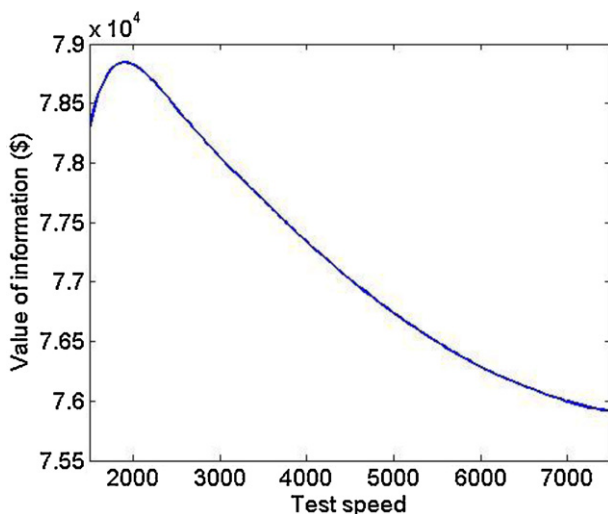


Fig. 10. Value of information for the first test. The maximum value of information is obtained at 1900 rpm.

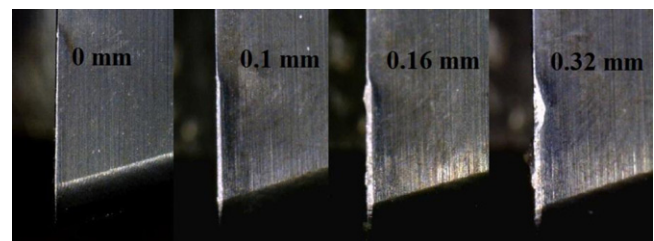


Fig. 13. Images of FWW at 60 $\times$  magnification. The cutting times from left to right are {0, 78.5, 166.4, and 255.3} min.

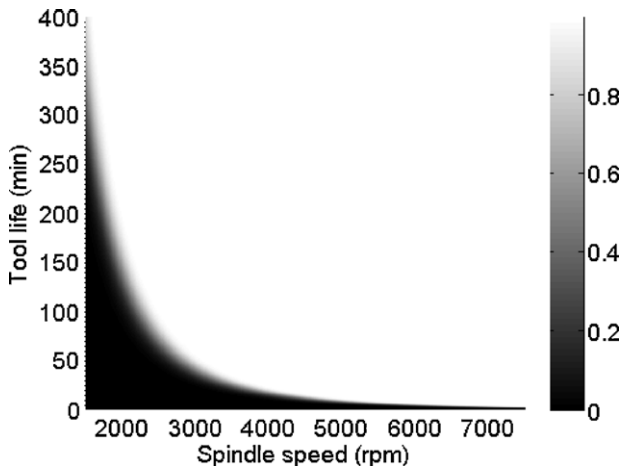


Fig. 14. Posterior cdf of tool life for a tool life of 164 min at 1900 rpm.

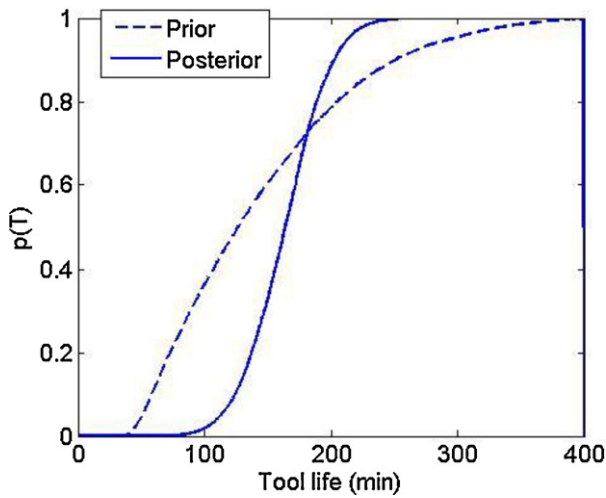


Fig. 15. Prior and posterior cdf of tool life at 1900 rpm.

Fig. 16 shows the value of information for the second test. According to this figure, the best test speed for the second test is 7500 rpm. A test was performed at 7500 rpm and a tool life of 3.2 min was obtained. The prior (which is the posterior after the

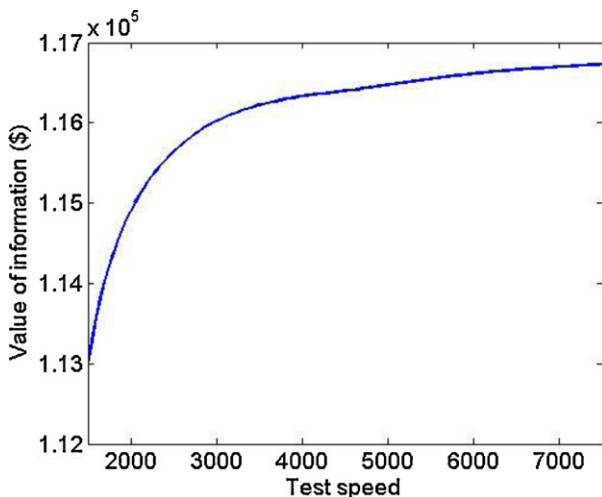


Fig. 16. Value of information for the second test. The maximum value of information is obtained at 7500 rpm.

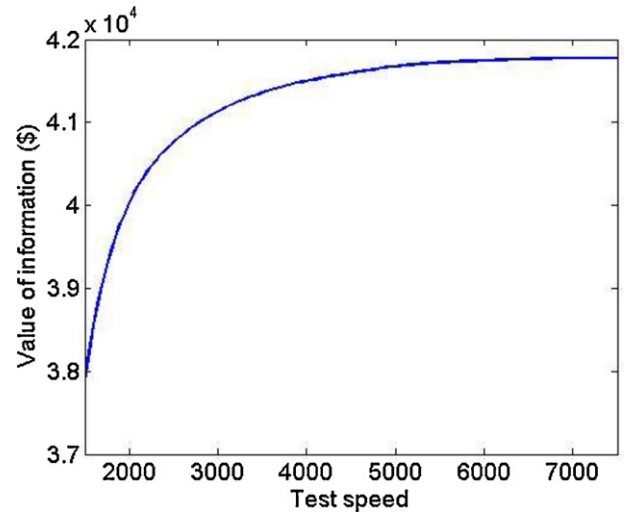


Fig. 17. Value of information for the third test. The maximum value of information is obtained at 7100 rpm.

Table 4

Test speeds and measured tool life for the three tests.

Test #	Test speed (rpm)	VOI (\$)	Tool life (min)
1	1900	78,847	164
2	7500	116,730	3.2
3	7100	41,777	3.7

first update) was again updated using the measured result. The procedure was repeated a third time to determine the next spindle speed for testing. Fig. 17 shows the value of information for the third test. The test speed was 7100 rpm and the measured tool life was 3.7 min. Table 4 summarizes the results. Fig. 18 shows the updated posterior tool life distribution after all three updates. Using this figure, the optimum tool life of 6.1 min was determined. The corresponding spindle speed is 5260 rpm and the expected cost is \$337.50.

### 5. Tool life predictions

The posterior tool life distribution can be used to predict tool life at spindle speeds other than the ones at which the tool wear experiments were performed. The posterior distribution was used to predict tool life at 2500 rpm and 5000 rpm. Three tests were per-

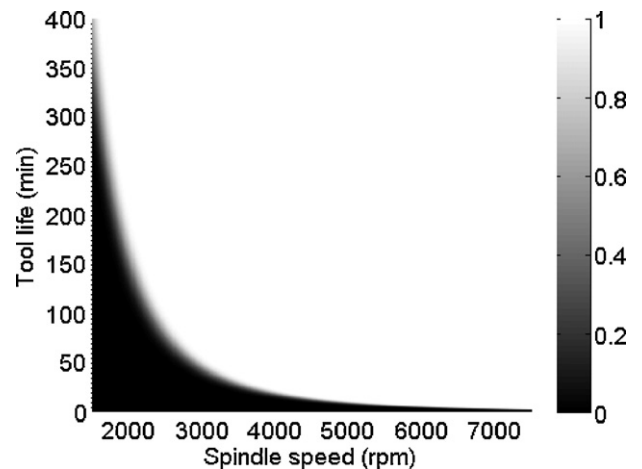
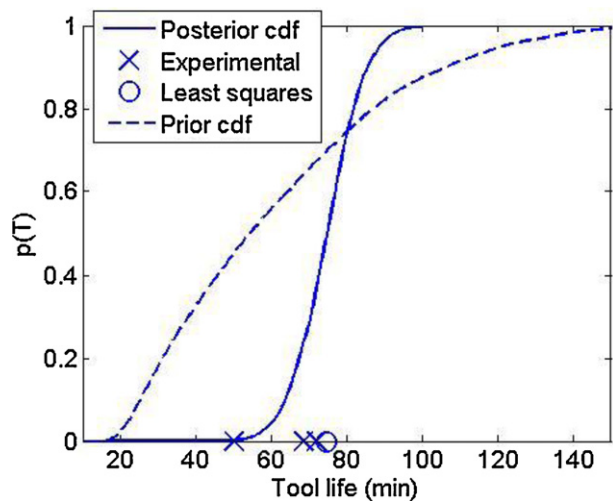


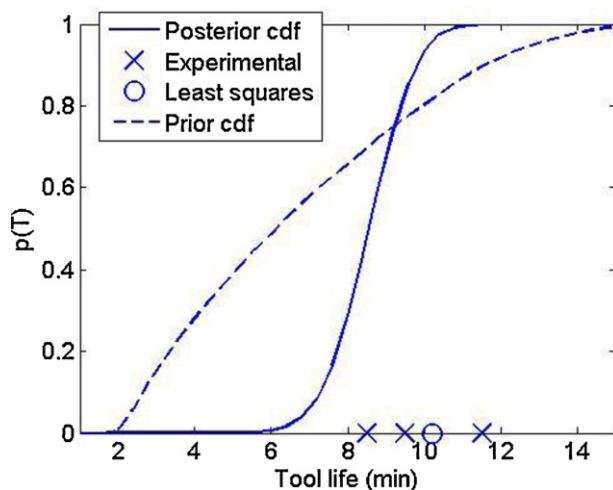
Fig. 18. Posterior cdf of tool life after three updates (see Table 4).

**Table 5**  
Experimental values of tool life for prediction.

Test #	Spindle speed (rpm)	Cutting speed (m/min)	Tool life (min)
1	2500	149.6	50.1
2	2500	149.6	68.5
3	2500	149.6	72.0
4	5000	299.2	11.5
5	5000	299.2	9.5
6	5000	299.2	8.5



**Fig. 19.** Tool life predictions at 2500 rpm using the posterior distribution and least squares fit.



**Fig. 20.** Tool life predictions at 5000 rpm using the posterior distribution and least squares fit.

formed at each spindle speed to compare against the predictions. The tests were performed at the same parameters as stated previously and the same procedure was followed to measure tool life. As before, tool life was the time for the tool to reach a FWW of 0.3 mm. Table 5 lists the experimental tool life values observed from the six tests. A least squares curve fit was completed using the data in Table 4 to identify the (deterministic) Taylor tool life equation constants. These values were  $n = 0.348$  and  $C = 621$  m/min (recall that the mean values from the literature search were 0.325 and 550 m/min). Using the curve fit  $n$  and  $C$  values, the tool life was predicted for the new spindle speeds. For comparison, the tool life cdf was evaluated using the prior (see Fig. 1). Figs. 19 and 20 display the posterior and prior distributions of tool life at 2500 rpm

and 5000 rpm, respectively, the experimental results ('x' symbols), and the deterministic predictions ('o' symbols).

To interpret the results, recall that the vertical axes of Figs. 19 and 20 represent the probability of tool failure and that the experimental results identify the tool life at which failure occurred (defined here as reaching a FWW of 0.3 mm). In Fig. 19, the Taylor tool life prediction coincides with a probability of failure of 0.5 from the posterior, so the two models agree well (assuming the deterministic value represents a tool life that can be reached half of the time). However, it is seen that: (1) the experiments yielded failure at posterior probabilities of 0.4 and lower; and (2) the measured tool life values were all less than the Taylor model prediction. Therefore, both models over-predicted tool life for this spindle speed. In Fig. 20, the Taylor tool life prediction coincides with a probability of failure of 0.95 from the posterior. In this case, the posterior is more conservative than the Taylor model prediction (which may be a preferred result, depending on the user's risk preference). The experiments show failure at probabilities from 0.5 to 1.0, so the posterior is also slightly more conservative than the test results indicate.

## 6. Conclusion

A Bayesian inference approach to tool life prediction was demonstrated using a random walk method. In Bayesian inference, a probability distribution is assigned over a range of the variable(s) of interest and the distribution is updated when new information is available. When this new information is obtained, uncertainty in the prior distribution can be reduced. Bayesian inference therefore provides a way to combine prior data with experimental values to update beliefs about an uncertain variable. Using the random walk approach, the prior probability of tool life was generated using sample tool life curves, where each path potentially represented the true tool life curve. The probability that each sample path was the true Taylor tool life curve was updated using Bayesian inference. A likelihood function was defined to describe how likely it was that the sample tool life curve was the correct curve given the measurement result at a particular spindle speed. An uncertainty of 20% was assumed for the measured tool life. The posterior tool life distribution was then used to predict the values of tool life at different spindle speeds and the results were compared to experiment.

A value of information approach was implemented to select the best experimental test speed(s). The value of information was defined as the expected cost prior to testing minus the expected cost after testing. The experimental test point was then chosen which added the most (expected) value to the profit. This approach combined Bayesian inference with decision analysis.

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