

Manufacturing and uncertainty

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Introduction

Goal

Select processing parameters to achieve the desired outcome for the manufactured product (properties, dimensions, function...)

Obstacles

- Requires input-output relationship between parameters and outcome
- Inputs are not perfectly known
- Input-output relationship includes approximations and omissions
- Outputs cannot be perfectly measured [1, 2]

Situation

- Establish input-output model (materials science, physics, chemistry...)
- Predict output(s) given input(s)
- Incorporate uncertainty

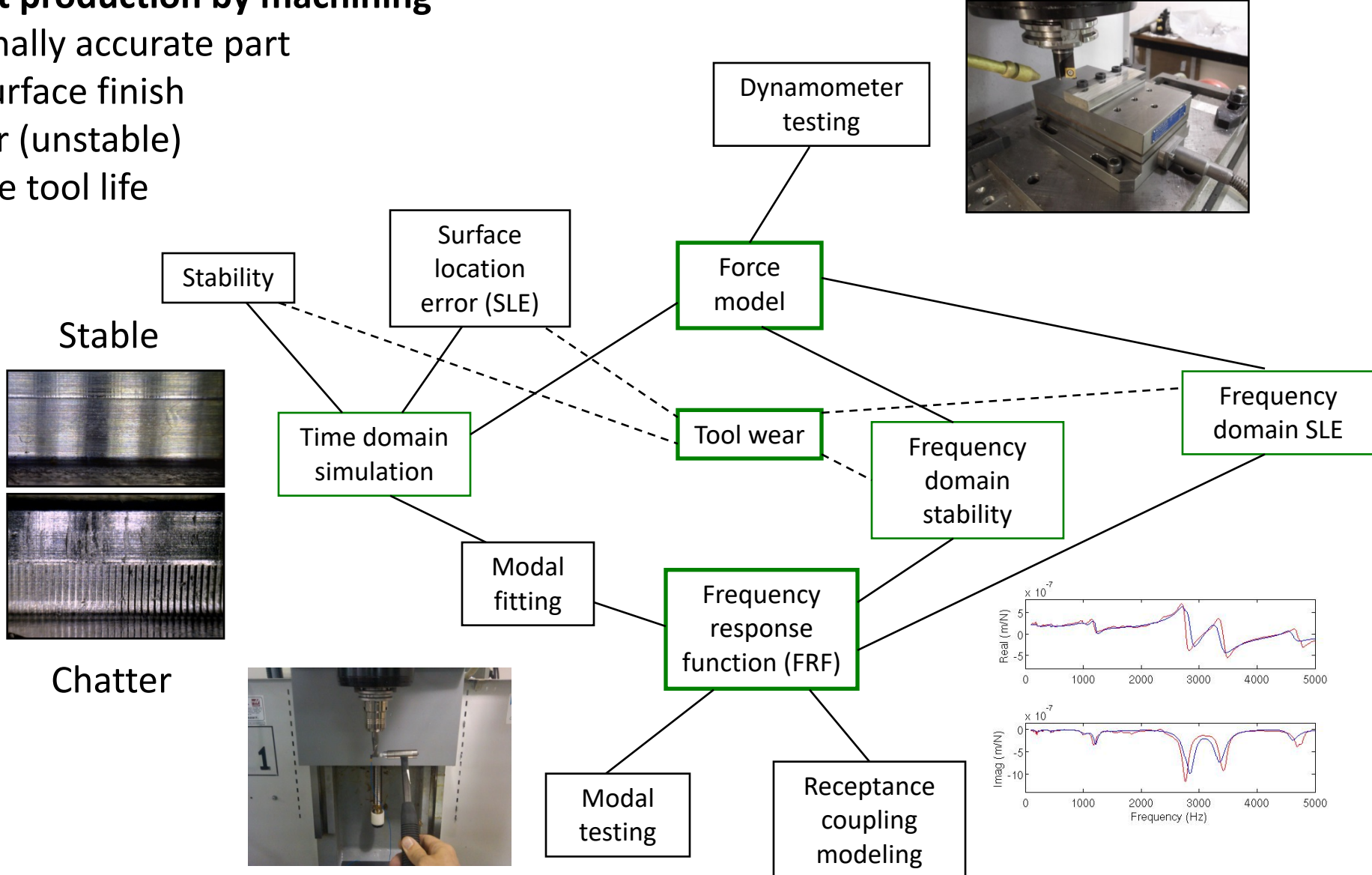
Consider machining as an example manufacturing operation.

1. ISO, 1993, Guide to the Expression of Uncertainty in Measurement, International Organization for Standardization, Geneva, Switzerland
2. Barry N. Taylor and Chris E. Kuyatt, 2001, Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, <http://physics.nist.gov/TN1297>, National Institute of Standards and Technology, Gaithersburg, MD.

Machining background

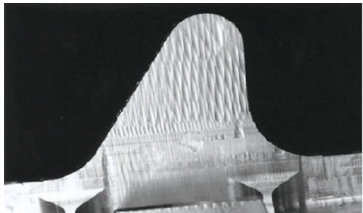
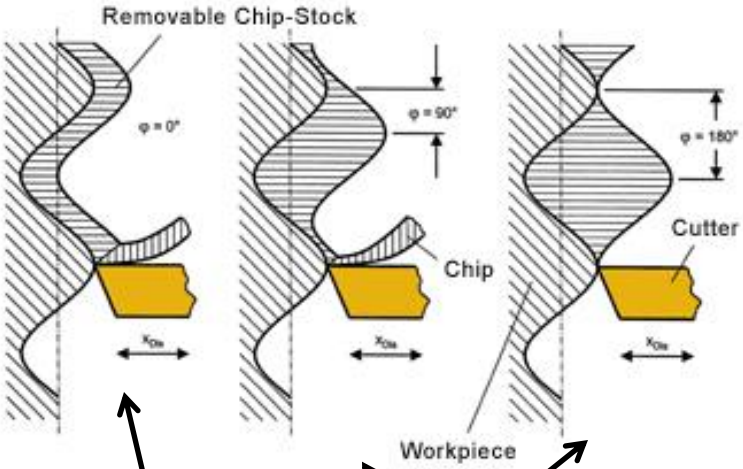
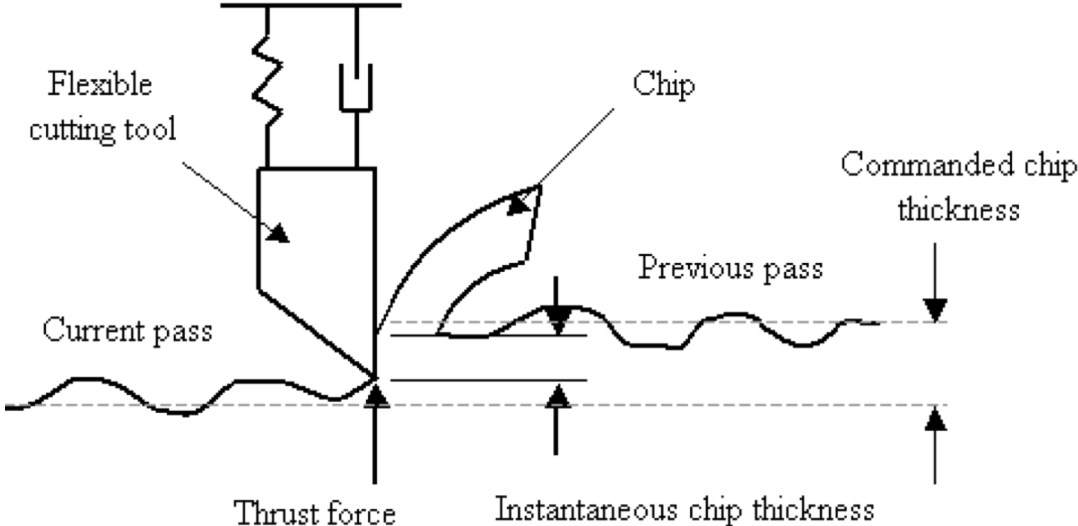
Goals for part production by machining

- Dimensionally accurate part
- Desired surface finish
- No chatter (unstable)
- Acceptable tool life



Machining background

Chatter – self-excited vibration that occurs in machining (large forces, poor finish)



Regeneration is a primary mechanism for chatter

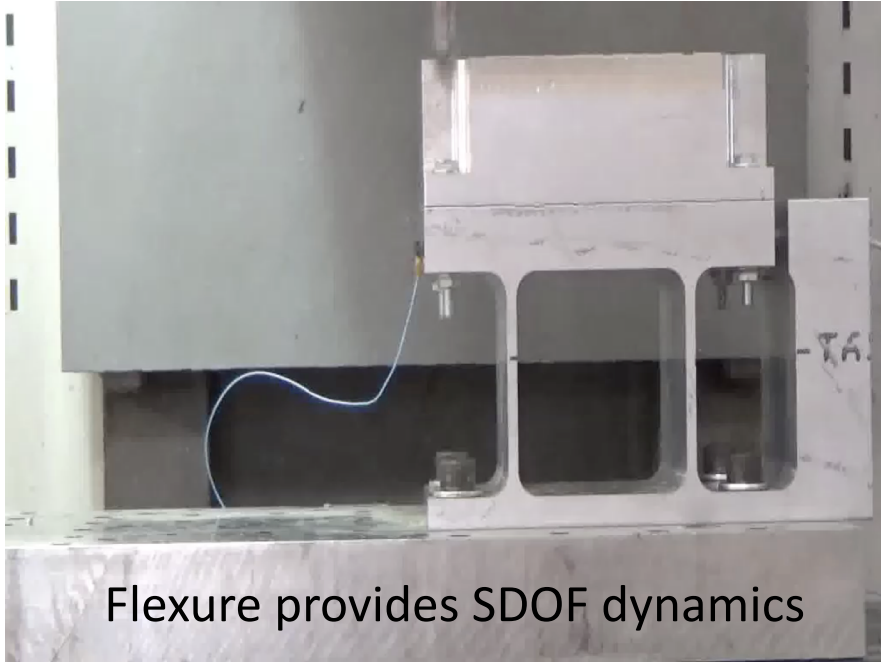
- force depends on chip thickness
- chip thickness depends on current vibration and previous pass
- current vibration depends on force

Chip thickness varies so force varies → unstable

Chip thickness is nearly constant – small force variation → stable

Machining background

Stable and unstable (chatter) milling examples

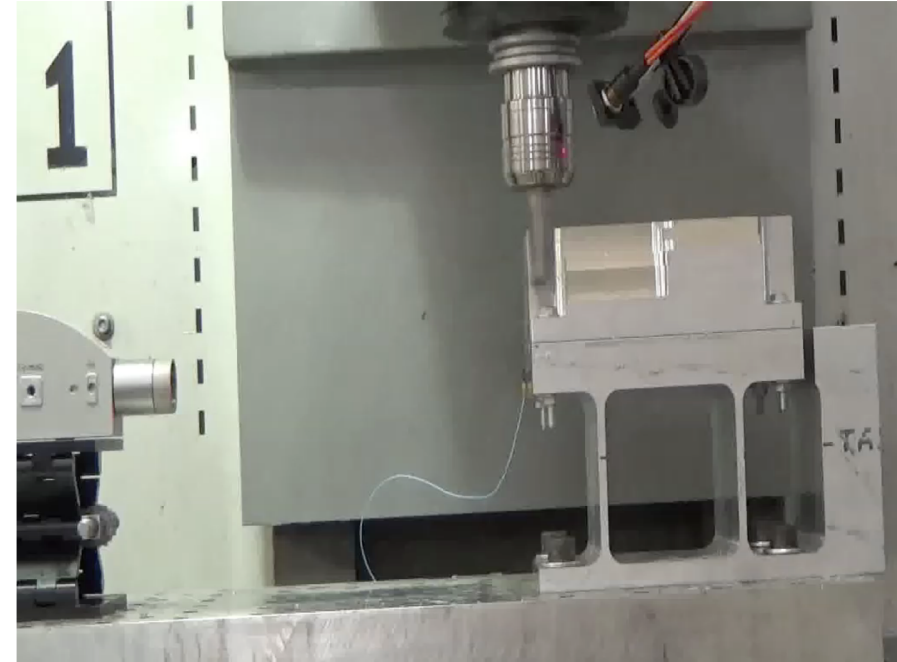


Stable:

Forced vibration

Repeats with each tooth passage

Tooth passing frequency and multiples



Chatter:

Self-excited vibration

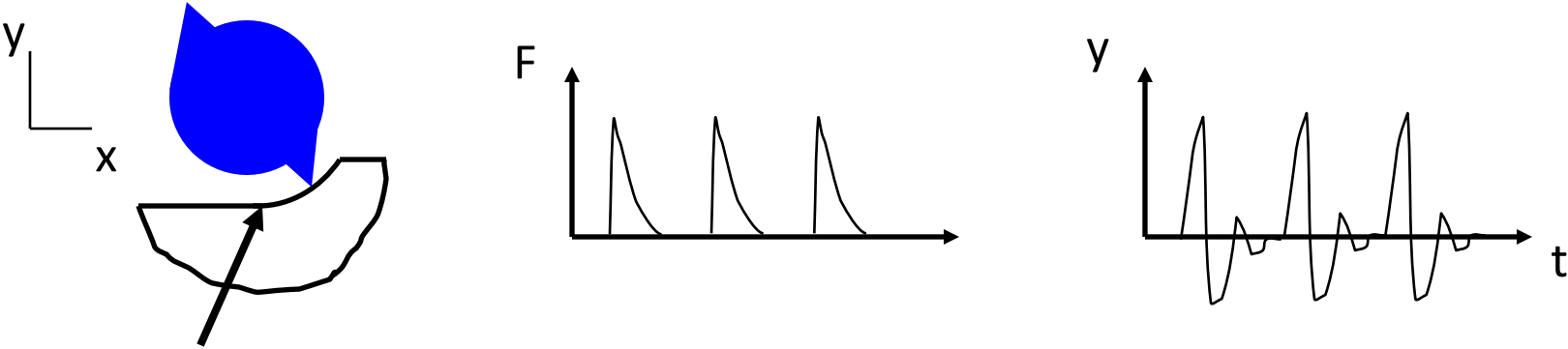
Does not repeat each tooth passage

Natural frequency of structure

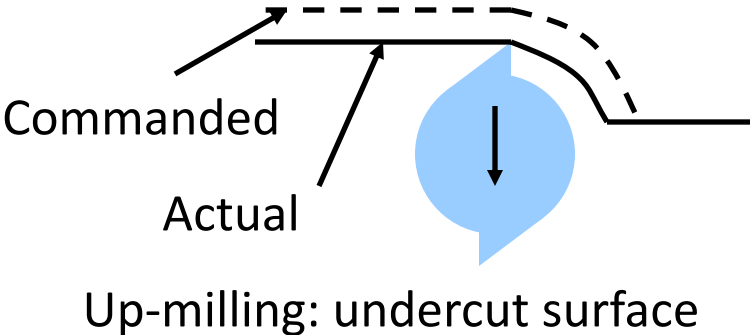
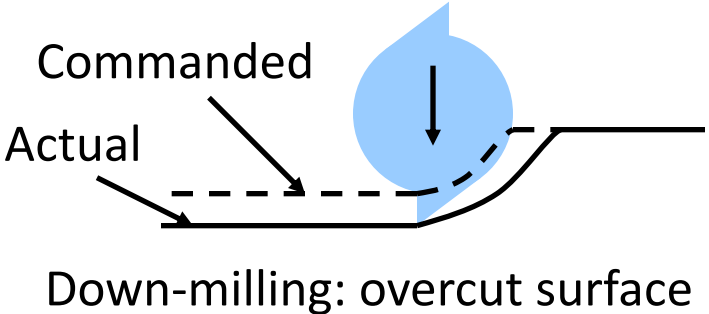
Machining background

Forced vibration during stable cutting can lead to **surface location error**

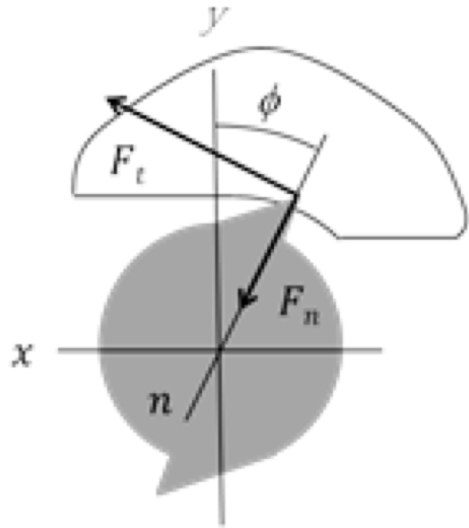
- vibration state of tool when leaving surface defines location
- magnitude and phase of vibration is frequency dependent (tooth passing frequency or spindle speed).



Where is the tool in its vibration cycle when it leaves the surface?



Milling description



Milling – a rotating cutter is used to remove material and leave the desired part geometry.

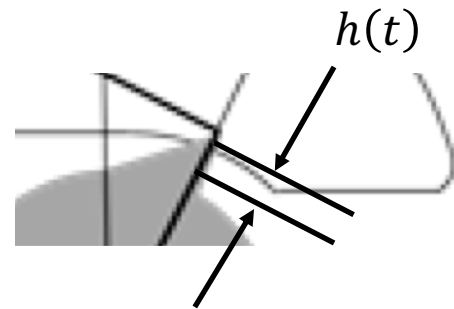
Vibration normal to the cut surface: $n(t) = x(t) \sin \phi(t) - y(t) \cos \phi(t)$

Chip thickness:

$$h(t) = \underbrace{f_t}_{\text{Feed per tooth}} \sin \phi(t) + \underbrace{n(t - \tau)}_{\text{Tooth period}} - n(t)$$

Feed per tooth

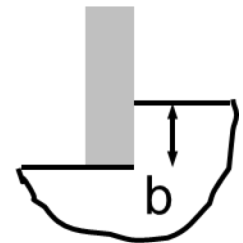
Tooth period



Time-delay term gives feedback (memory) in x and y

Force components: $F_t = k_{tc}bh(t) + k_{te}b$ $F_n = k_{nc}bh(t) + k_{ne}b$

Axial depth of cut



Project into x/y directions:

$$F_x(t) = F_t \cos \phi(t) + F_n \sin \phi(t) \quad F_y(t) = F_t \sin \phi(t) - F_n \cos \phi(t)$$

Milling description

System dynamics are described by a set of **second order time-delay differential equations**.

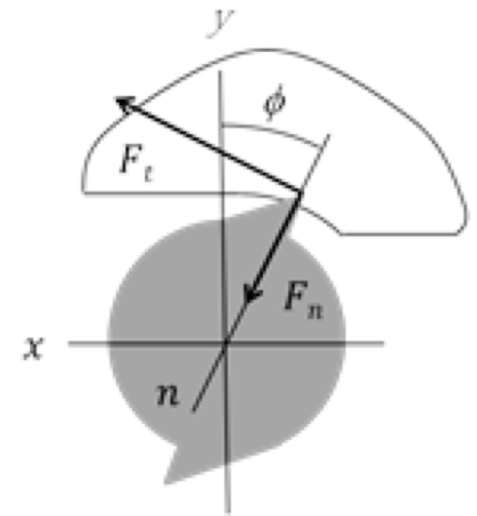
$$\begin{aligned} m_x \ddot{x}(t) + c_x \dot{x}(t) + k_x x(t) &= F_x(t) \\ m_y \ddot{y}(t) + c_y \dot{y}(t) + k_y y(t) &= F_y(t) \end{aligned}$$

Include x and y time-delay terms.

Describe tool/workpiece mass, damping, and stiffness in x/y directions.

Closed-form solution for set of delay differential equations is not available. Solution techniques include:

- analytical – approximate solution used to determine stability limit as a function of operating parameters (spindle speed, axial depth of cut)
- numerical – time domain simulation.



Time domain simulation

Solve set of second order time-delay differential equations using numerical integration.

Simulation steps per tooth period $S = \frac{2\pi}{N_t \cdot d\phi}$

$$n(i) = x(i-1) \sin \phi(i) - y(i-1) \cos \phi(i)$$

$$h(i) = f_t \sin \phi(i) + n(i - S) - n(i)$$

$$F_t = k_{tc} b h(i) + k_{te} b \quad F_n = k_{nc} b h(i) + k_{ne} b$$

$$d\phi \quad F_x(i) = F_t \cos \phi(i) + F_n \sin \phi(i)$$

$$F_y(i) = F_t \sin \phi(i) - F_n \cos \phi(i)$$

$$\ddot{x}(i) = \frac{F_x(i) - c_x \dot{x}(i-1) - k_x x(i-1)}{m_x}$$

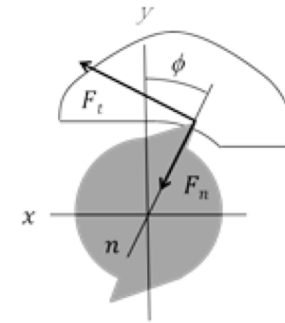
$$\ddot{y}(i) = \frac{F_y(i) - c_y \dot{y}(i-1) - k_y y(i-1)}{m_y}$$

$$\dot{x}(i) = \dot{x}(i-1) + \ddot{x}(i) \cdot dt$$

$$\dot{y}(i) = \dot{y}(i-1) + \ddot{y}(i) \cdot dt$$

$$x(i) = x(i-1) + \dot{x}(i) \cdot dt$$

$$y(i) = y(i-1) + \dot{y}(i) \cdot dt$$

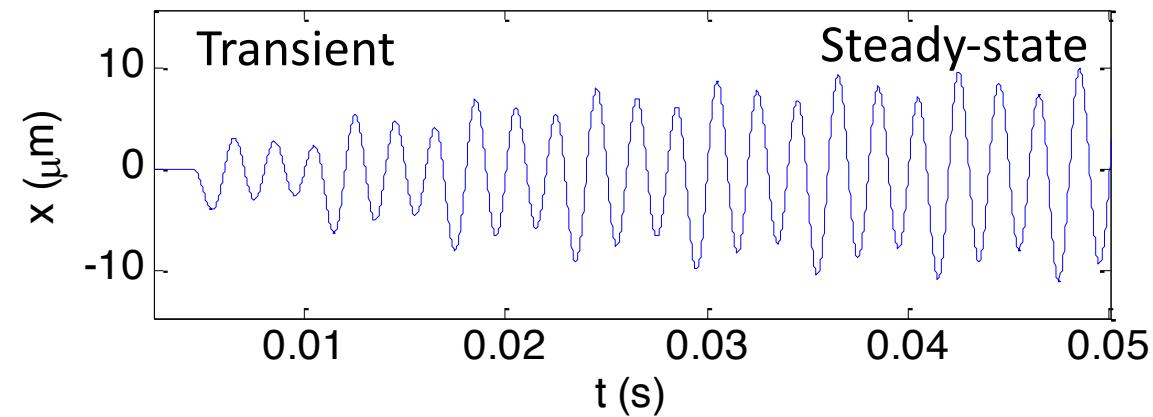
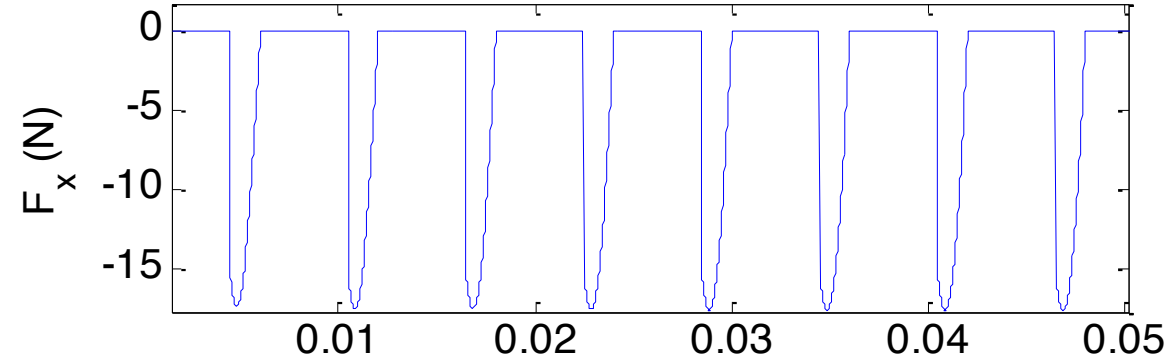


Time domain simulation

Simulation
inputs

- Cutting conditions: spindle speed, radial/axial depth, feed per tooth, cutting force coefficients.
- Tool geometry: number of teeth, diameter, helix angle.
- Tool point modal parameters: m , c , k in the x and y directions.

Simulation
outputs



Bifurcation prediction

For dynamic systems, a *bifurcation* is a dramatic change in the system state, or behavior.

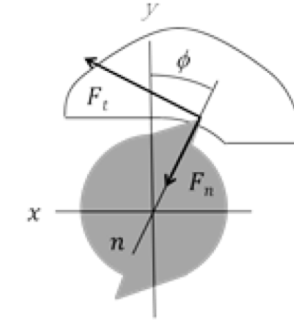
Milling exhibits various bifurcation (instability) types.

- A powerful interrogation tool for milling dynamics is **periodic sampling at the tooth period**.
- This sampling establishes the synchronicity of the motion (response) with the cutting force (excitation).
- For stable cutting conditions, only **forced vibration** is present and the sampled point repeats for each tooth passage (stable).
- For unstable cutting, on the other hand, the repetition of a single point is not observed and the character of the sampled points identifies the type of instability (chatter): secondary **Hopf** or period-n **bifurcations**.

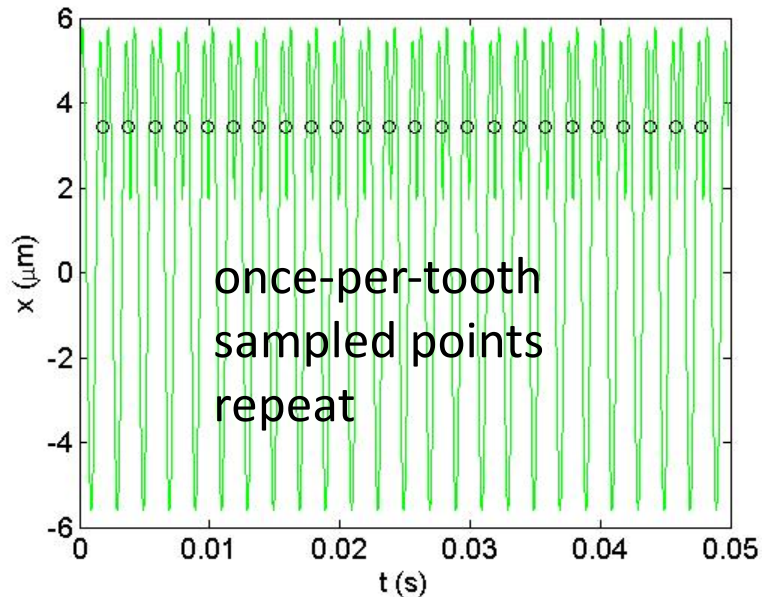
Bifurcation prediction

Example

- 5% radial immersion up milling
- 30000 rpm spindle speed
- 721 Hz natural frequency, 0.009 damping ratio, and 4.1×10^5 N/m stiffness
- cutter has one tooth, a 45 deg helix angle, and an 8 mm diameter
- aluminum alloy cutting force coefficients are: $k_{tc} = 604 \times 10^6$ N/m² and $k_{nc} = 223 \times 10^6$ N/m² (zero edge coefficients)

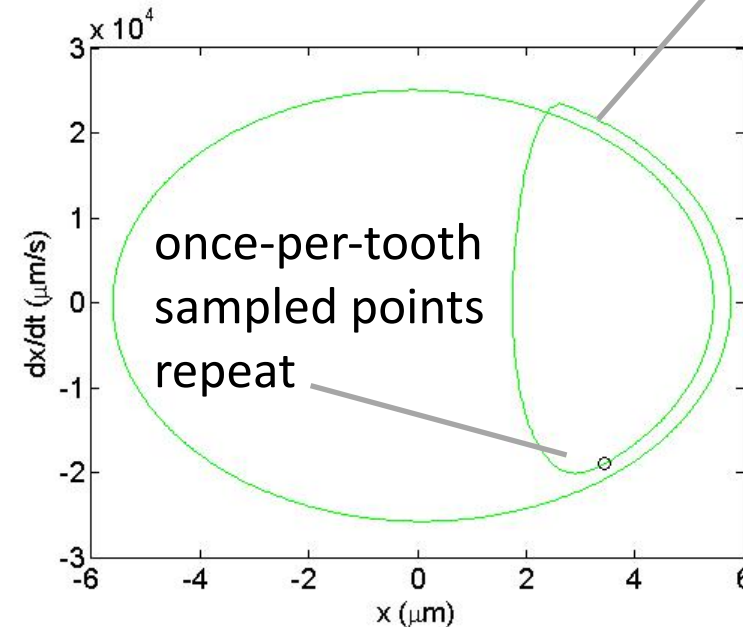


Stable cut, $b = 0.5$ mm



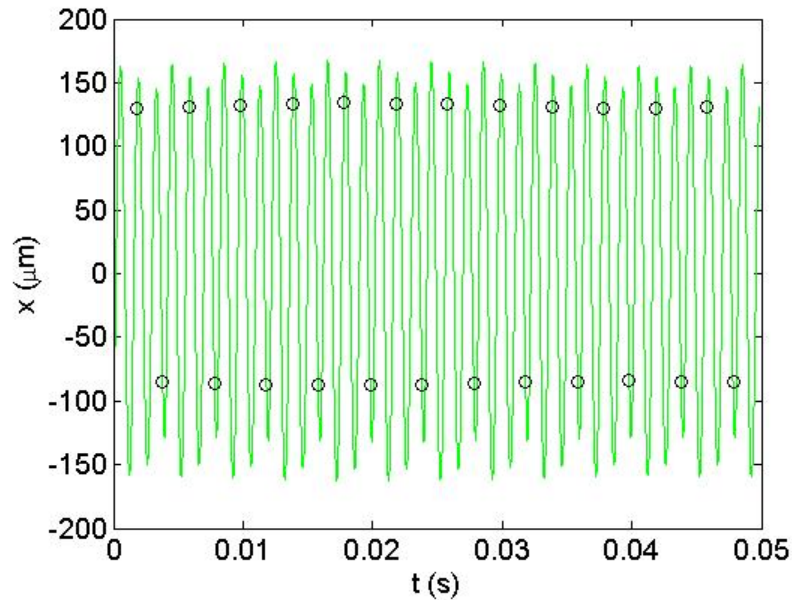
Poincaré map

x vs. \dot{x}

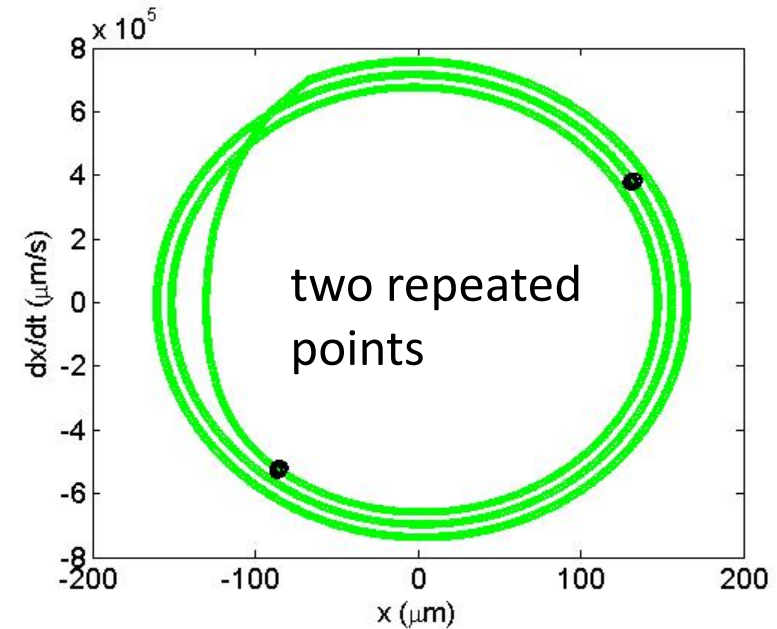


Bifurcation prediction

Unstable cut, $b = 2.5$ mm



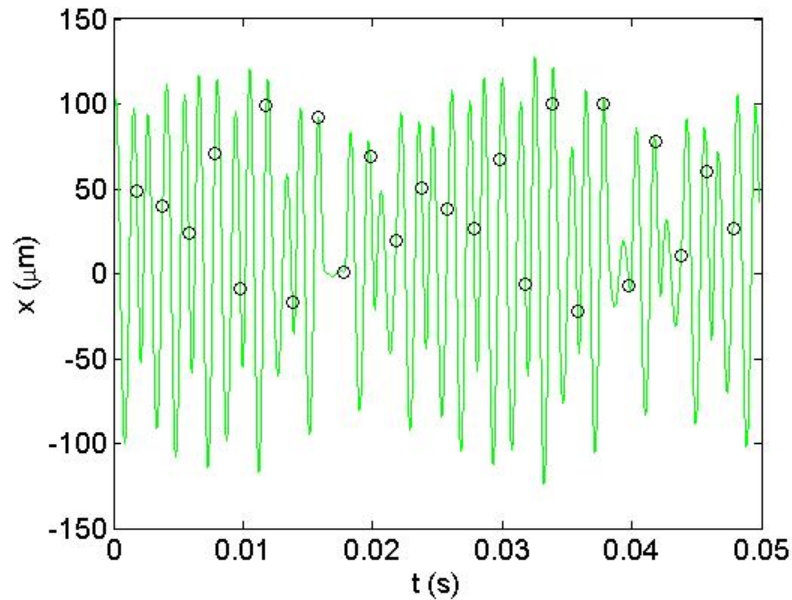
Poincaré map



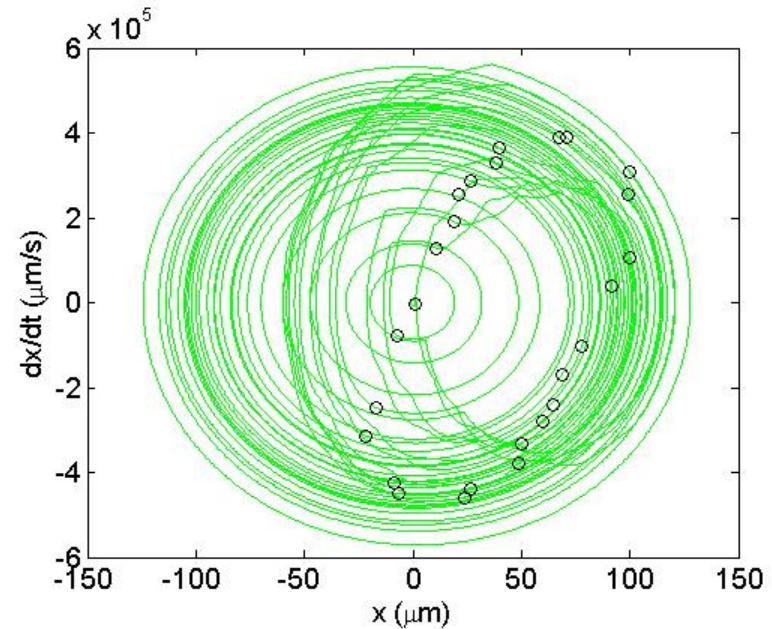
Period-2 bifurcation – once-per-tooth sampled points repeat at two distinct locations (special type of instability or chatter).

Bifurcation prediction

Unstable cut, $b = 5$ mm



Poincaré map



Secondary Hopf bifurcation – once-per-tooth sampled points do not repeat.

Chatter frequency is near the system natural frequency. This incommensurate frequency yields an **elliptical distribution** of points in the Poincaré map.

Results

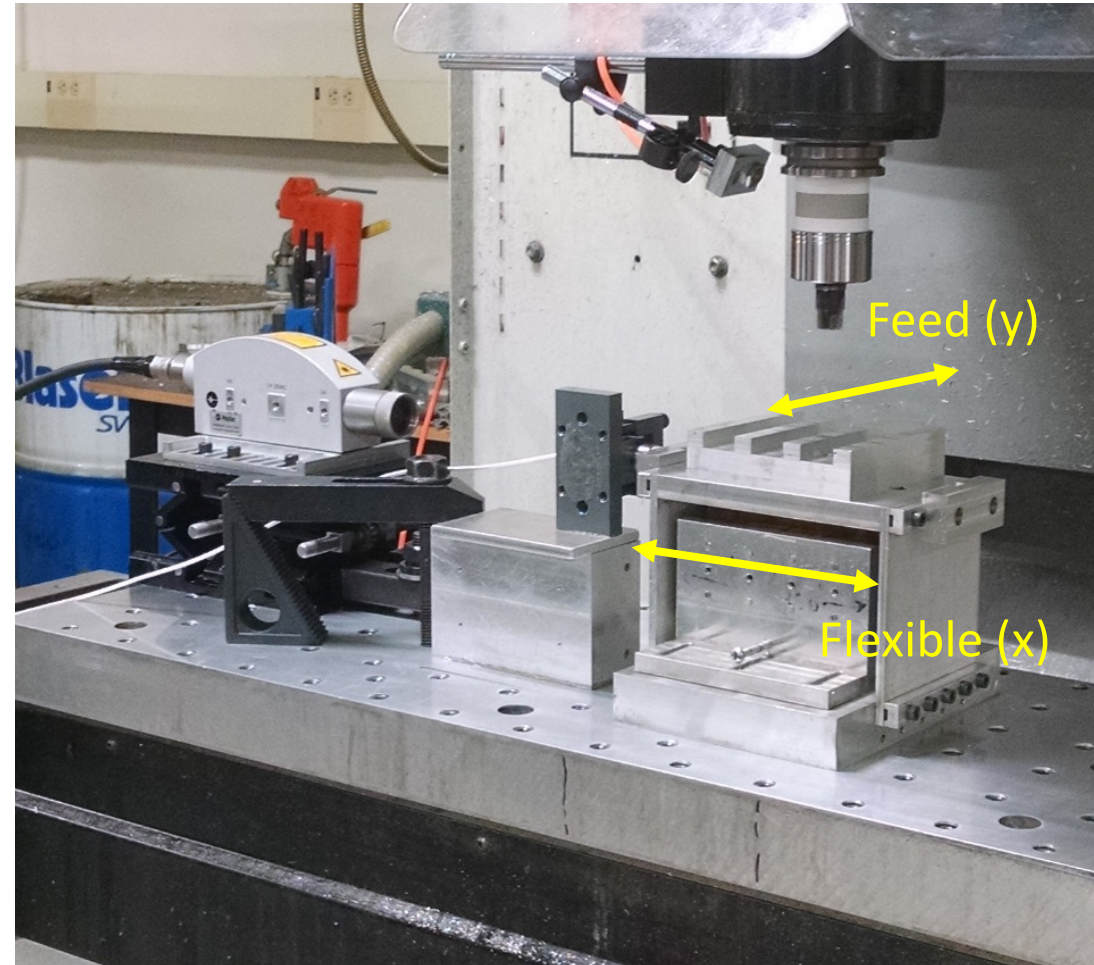
Experimental setup for stability and SLE

Flexure dynamics

- Stiffness: 1.75×10^6 N/m
- Damping ratio: 1.36%
- Natural frequency: 125.8 Hz

Tool dynamics

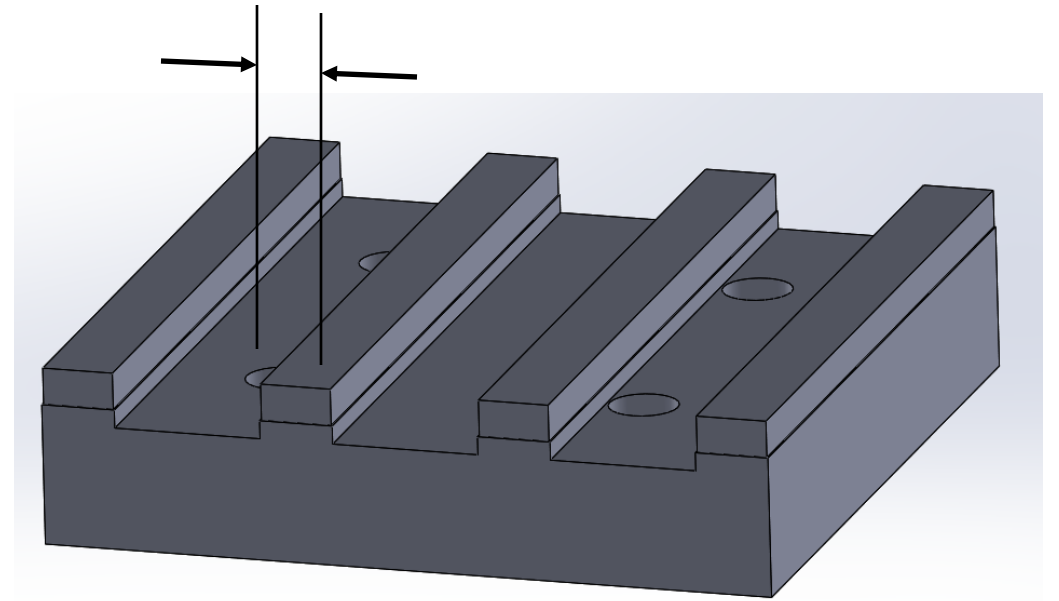
- Stiffness: 4.24×10^7 N/m
- Damping ratio: 9.5%
- Natural frequency: 1188 Hz



Results

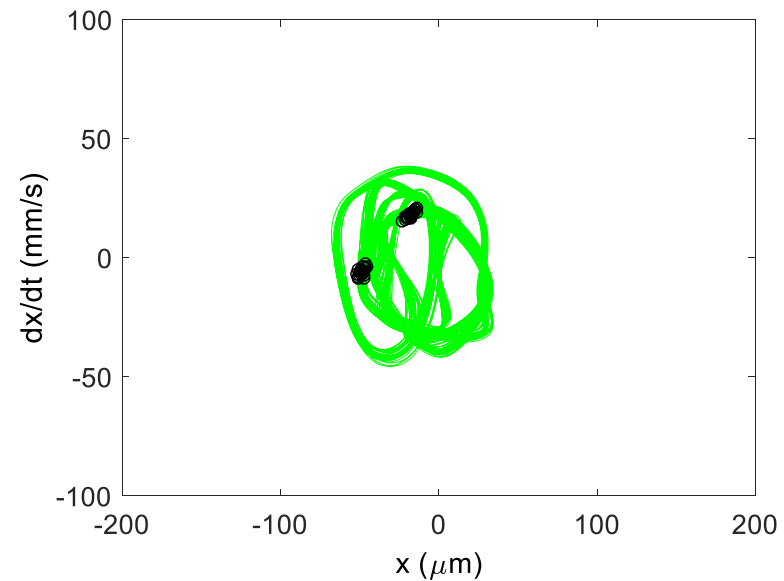
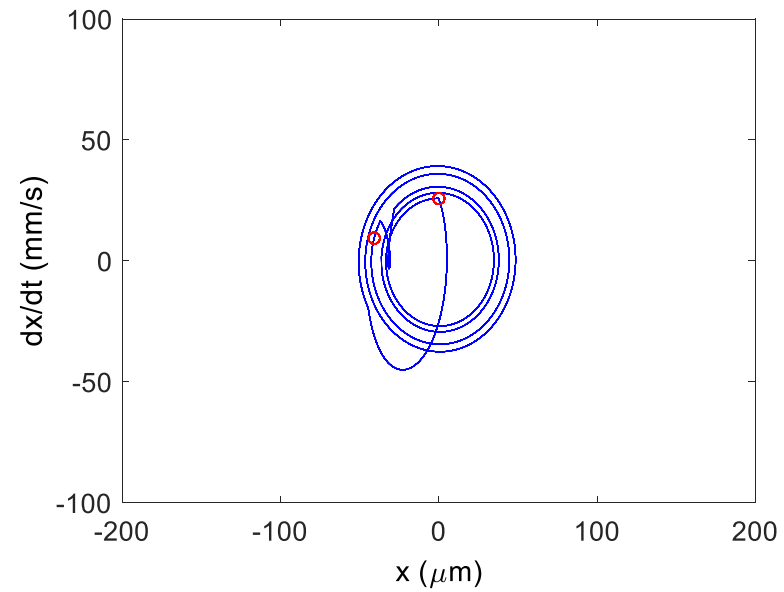
- Initial ribs machined on flexure (9.82 mm wide).
- Final pass completed with 2 mm radial depth of cut, 5 mm axial depth of cut.
- Spindle speed was varied.
- 0.35 mm/tooth
- Up milling
- Single carbide insert cutter
- 6061-T6 aluminum workpiece
- Surface location error (SLE) was measured.

SLE = commanded width – actual width



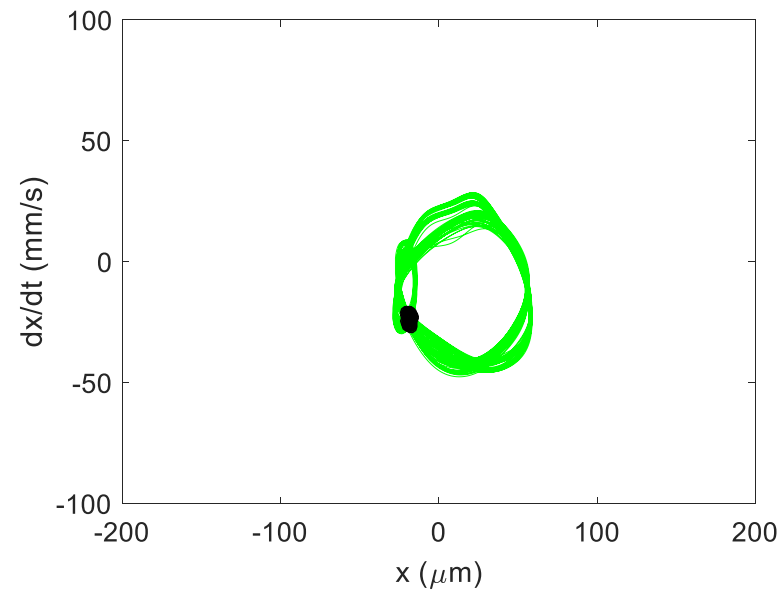
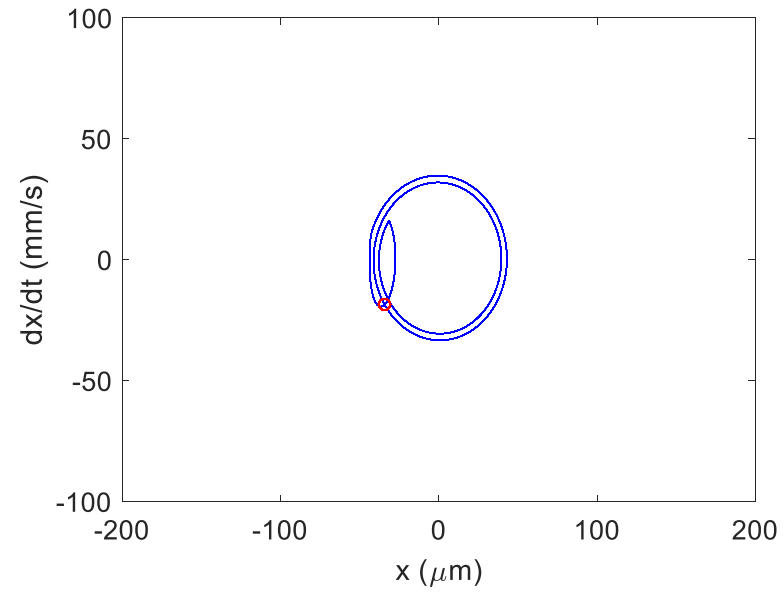
Results

| Spindle speed (rpm) | Behavior |
|---------------------|-----------------|
| 3180 | Period-2 |
| 3190 | Period-2 |
| 3200 | Period-2 |
| 3210 | Period-2 |
| 3270 | Stable |
| 3300 | Stable |
| 3330 | Stable |
| 3360 | Stable |
| 3400 | Stable |
| 3500 | Stable |
| 3600 | Stable |



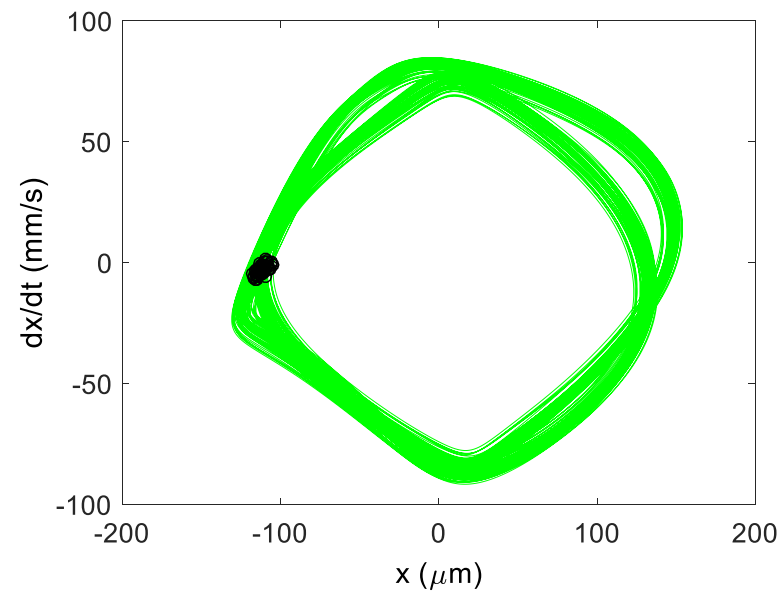
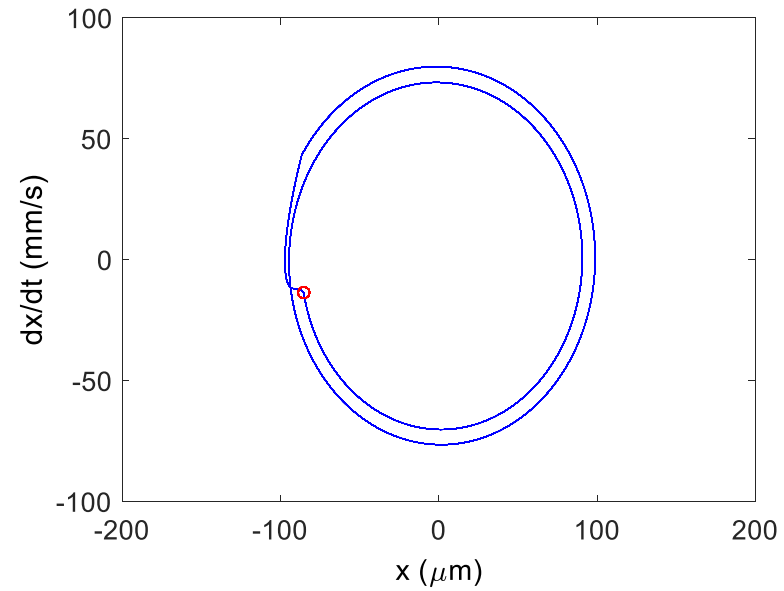
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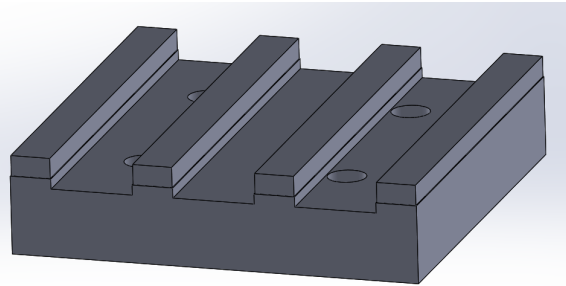


Results

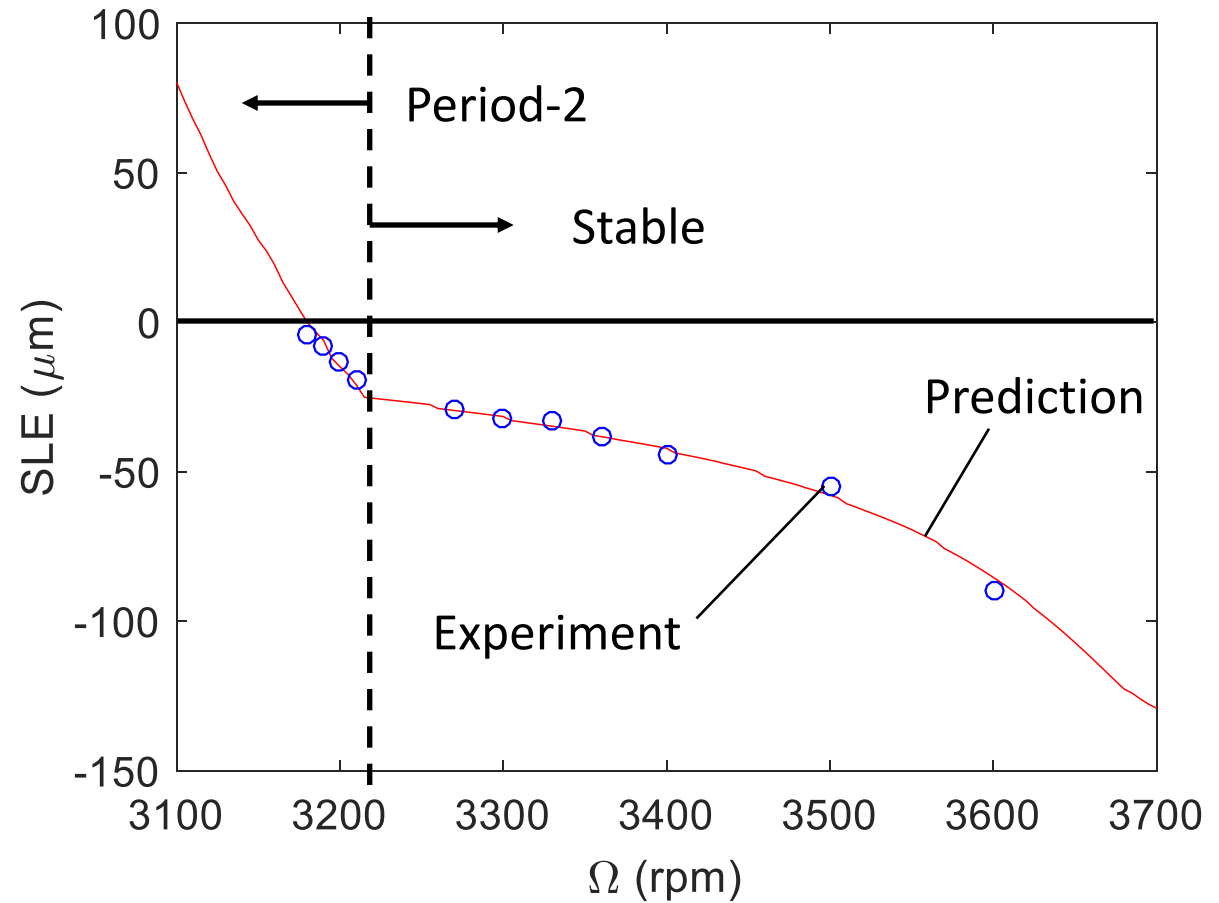
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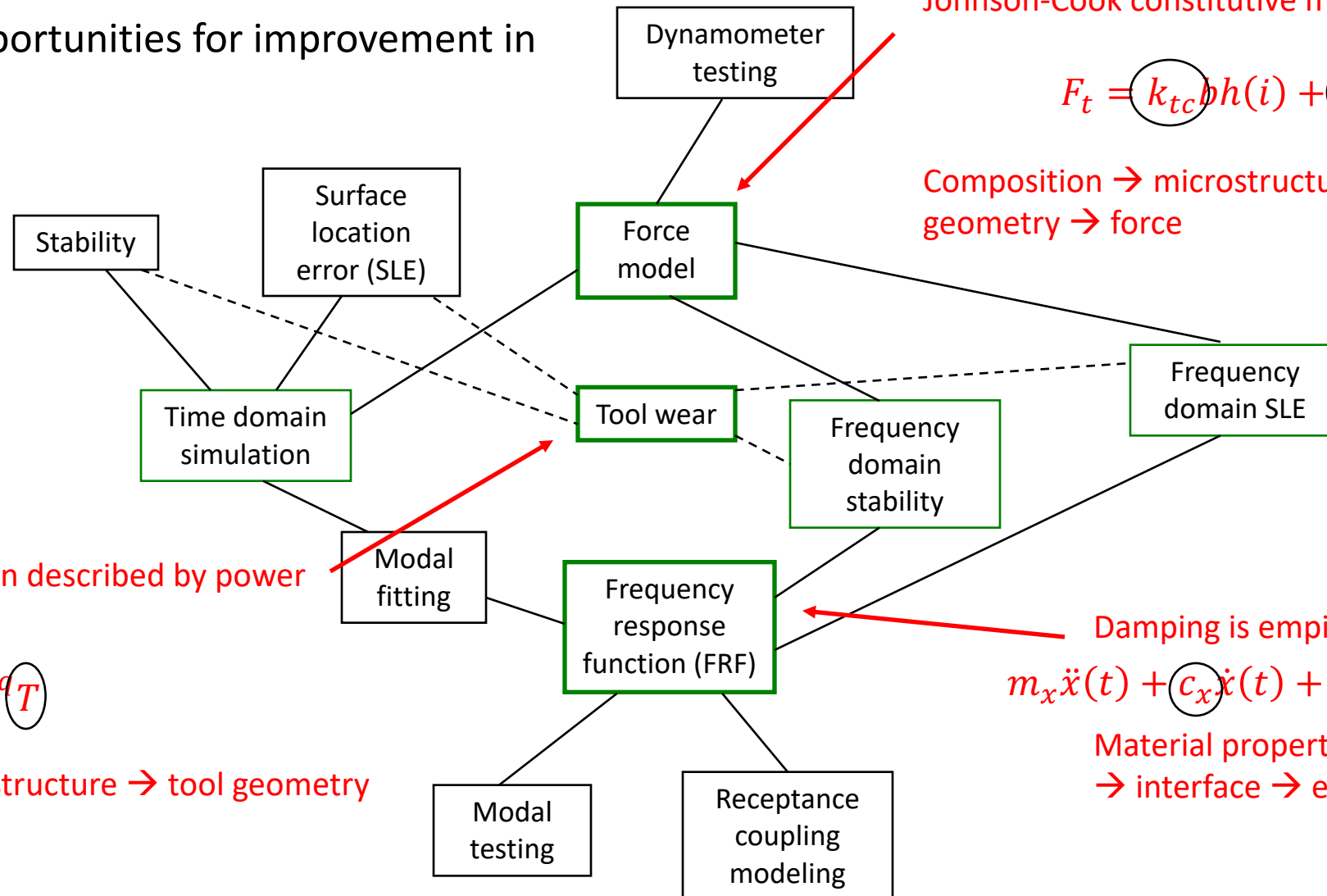


- Parts were measured on CMM and SLE was calculated.
- Experimental results compared to prediction.



Opportunities

- Predictions and experiments differ due to uncertainties.
- What are the opportunities for improvement in machining?



Typically empirical. Can be described using Johnson-Cook constitutive model (or similar).

$$F_t = k_{tc} b h(i) + k_{te} b$$

Composition → microstructure → tool geometry → force

Empirical. Tool life often described by power law (Taylor 1906).

$$C = v^p f_t^q T$$

Composition → microstructure → tool geometry → wear rate

Damping is empirical.

$$m_x \ddot{x}(t) + c_x \dot{x}(t) + k_x x(t) = F_x(t)$$

Material properties → geometry → interface → energy dissipation

Opportunities

Similar opportunities available for other manufacturing operations.

Requirements:

- Process knowledge to define first-principles models (or AI?)
- Materials modeling to relate alloy composition to process behavior
- Experimental capabilities to validate models
- Propagation of input uncertainty to output uncertainty (numerical or analytical)
- Parameter selection under uncertainty (optimization)

Thank you.

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