COMPARISON OF ANALYTICAL MILLING STABILITY ANALYSES WITH TIME DOMAIN SIMULATION

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INTRODUCTION

In the past few decades, machine and cutting tool technology has increased in both capability and complexity. Despite the significant advancements, however, self-excited vibrations (or chatter) remain a limiting factor for material removal rates and part quality.

In 1965, Tobias showed that chatter is self-excited vibration which results from regeneration effects on instantaneous chip thickness [1]. The cutting edge removes a chip that was produced by in the previous pass (the prior revolution in turning or tooth in milling). The chip thickness, which affects the force and therefore the vibration response, depends on the phase between the previously cut surface and the current vibration. This understanding led to the development of analytical algorithms which are used to generate a stability map of the limiting axial depth of cut, b_{lim}, versus spindle speed, Ω. This map is known as a stability lobe diagram. As an alternative, the governing equations of motion may be solved in the time domain to define the machining behavior (time domain simulation).

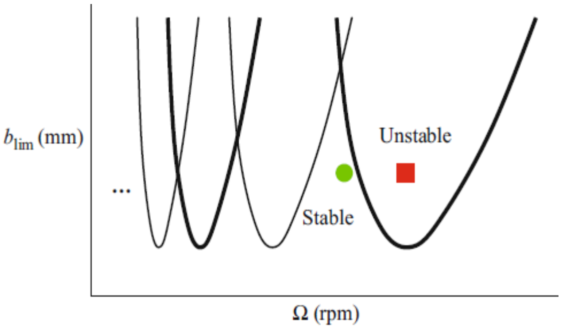


Figure 1. Stability lobe diagram [2].

In this paper, two popular frequency domain stability analyses are compared to time domain simulation results. A new stability metric is introduced to automatically identify the time domain simulation stability.

FREQUENCY DOMAIN STABILITY ANALYSIS

Frequency domain stability analysis is used to determine the global stability of a milling operation as a function of spindle speed and depth of cut. Two popular frequency domain techniques are:

1. average tooth angle approach developed by Tlusty [3]
2. Fourier series approach developed by Altintas and Budak [4].

The average tooth angle approach assumes an average tooth angle in the cut (the mean of the entry and exit angles) and, therefore, an average force direction. This assumption creates an autonomous, or a time-invariant, system. Tlusty then made the use of directional orientation factors to first project this average force into the x (feed) and y-directions and, second, project these results onto the surface normal.

The Fourier series approach transforms the dynamic milling equations into a time-invariant, but radial immersion-dependent, system. The frequency domain algorithm accounts for the x and y-direction tool deflections and uses a truncated Fourier series (mean value only) to represent the cutting force.

TIME DOMAIN STABILITY ANALYSIS

Time domain simulation is used to calculate the forces and displacements during milling. It gives local, rather than global, information about the process behavior for a given spindle speed and axial depth of cut. The simulation implemented in this study [2] is based on the “Regenerative Force, Dynamic Deflection” model described by Smith and Tlusty [5]. The simulation proceeds as follows.

1. The instantaneous chip thickness is determined using the vibration of the previous and current teeth at the selected tooth angle.

2. The cutting force components are calculated as the product of the chip width (axial depth), chip thickness, and specific force coefficients.
3. The force is used to find the new displacement by Euler (numerical) integration of the second order, time delay equations of motion.
4. The tooth angle is incremented and the process is repeated.

The simulation takes into account the non-linearity that can occur if the tooth leaves the cut due to large vibration amplitude. It can also accommodate cutter teeth runout, variable teeth spacing, and variable helix angles.

To automatically identify stable or unstable behavior from the simulated displacements, a new stability metric was defined. The new metric is: $\mathbf{M} = \sum_{n=1}^{\mathbf{N}} \left|\frac{(\mathbf{xts(i)}-\mathbf{xts(i-1)})}{N}\right|$, where where xts is the vector of once-per-tooth sampled tool displacements in the x-direction and N is the number of points. Similarly, this metric can also be applied to the y-direction tool displacements or the workpiece displacements. The absolute value of the difference in the successive sampled points is summed. If the cut is stable (forced vibrations), the sampled points repeat and the M value is nominally zero. In this research, M < 1 µm was used to identify a stable cut, while an M value greater than or equal to 1 µm indicated an unstable cut.

RESULTS

To compare the frequency and time domain analyses, the system dynamics were selected and the stability was identified over a range of spindle speeds and axial depths of cut. As an example, the system dynamics for a single degree of freedom (DOF) system were specified as shown in Table 1.

Table 1. Single DOF dynamics, flexible tool.

Tool dynamics	**x**	**y**
Natl. freq. (Hz)	750	750
Damping (%)	5	5
Stiffness (N/m)	5×10^6	5×10^6
Workpiece dynamics	**x**	**y**
Natl. freq. (Hz)	750	750
Damping (%)	5	5
Stiffness (N/m)	5×10^{10}	5×10^{10}

The diameter of the two-flute milling cutter was 20 mm in all cases. The specific cutting force for the aluminum alloy workpiece was 700 x10^6 N/m^2 and the force angle was 68 deg [2].

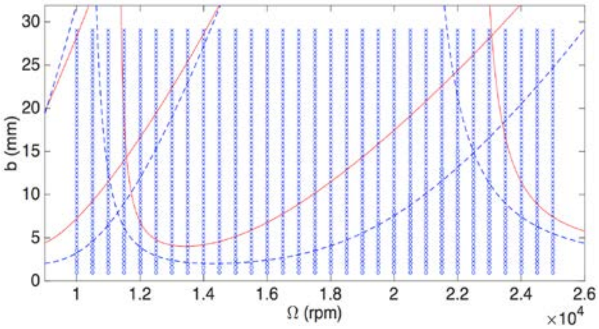
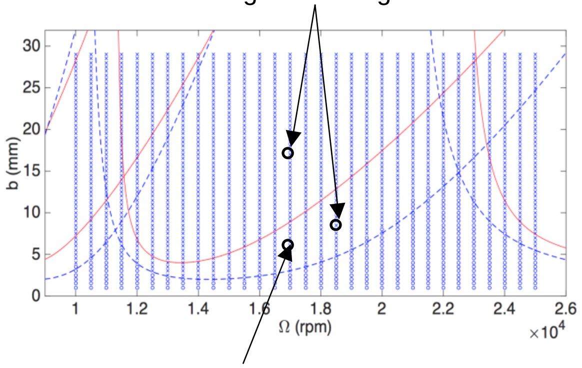


Figure 2. Comparison of frequency and time domain stability results.

Figure 2 shows the two frequency domain solutions superimposed over a grid of time domain simulation results, where the solid line represents the average tooth angle approach, the dotted line represents the Fourier series approach, a circle indicates a stable time domain simulation result, and an × identifies an unstable result. The up milling radial immersion used to generate this figure was 50%.

Stable points (o) are correct for Fourier series and average force angle.



Unstable point (×) is correct for Fourier series, but incorrect for average force angle.

Figure 3. Procedure for counting correct and incorrect grid points to determine accuracy of frequency domain stability analyses.

To quantify the accuracy of the frequency domain stability analyses, the number of incorrect points was tallied for both approaches and the percent correct score was calculated. For each point in the simulation grid, the stability was determined (time domain simulation was assumed to be correct). If an unstable point was below the frequency domain stability limit, the point was considered incorrect. Similarly, if a stable point occurred above the frequency domain limit, it was identified as incorrect. Figure 3 demonstrates the counting procedure graphically. This figure has 1767 time domain simulation points in the grid. There are 378 incorrect points for the average tooth angle approach, which corresponds to a percent correct score of 78.6%. The Fourier series approach has 25 incorrect points, which gives a 98.6% correct score.

This procedure was repeated for multiple cases, including both up milling and down milling, different radial immersions, and varying system dynamics:

1. single DOF symmetric tool (Table 1)
2. single DOF asymmetric tool
3. two DOF symmetric tool
4. two DOF a symmetric tool
5. single DOF symmetric (equal tool-workpiece)
6. single DOF symmetric (unequal tool-workpiece).

A special low damping case for single DOF tool dynamics was also considered. To compare the two frequency domain analyses, the average percent correct score for all six dynamic cases is provided in Tables 2 and 3 as a function of radial immersion (100% is slotting) for both up and down milling.

Table 2. Comparison of up milling percent correct scores from all six dynamic cases.

Radial immersion	Frequency domain analysis	Average score (%)
100%	Fourier series	96.8
	Average tooth angle	77.0
75%	Fourier series	96.6
	Average tooth angle	77.8
50%	Fourier series	96.9
	Average tooth angle	82.1
25%	Fourier series	95.8
	Average tooth angle	91.1
5%	Fourier series	93.9
	Average tooth angle	90.5

Table 3. Comparison of down milling percent correct scores from all six dynamic cases.

Radial immersion	Frequency domain analysis	Average score (%)
75%	Fourier series	95.2
	Average tooth angle	90.4
50%	Fourier series	97.3
	Average tooth angle	82.3
25%	Fourier series	95.3
	Average tooth angle	90.3
5%	Fourier series	94.3
	Average tooth angle	91.5

Tables 2 and 3 summarize the average percent correct score for the six system dynamics cases at each radial immersion. It is observed that the Fourier series approach more closely agrees with the time domain simulation results. The percent correct score for the average tooth angle approach tends to increase as the radial immersion decreases.

SINGLE DOF LOW DAMPING

A special case was also simulated where the tool was lowly damped and much more flexible than the workpiece. The system dynamics are summarized in Table 4. Time domain simulations were performed with low radial immersion (5%) for both up milling and down milling. The results are presented in Figs. 4 and 5. It is observed that neither frequency domain approach accounts for the new unstable zone between 14000 and 16000 rpm (period-2 bifurcation). This occurs due to the low damping and low radial immersion (highly interrupted cutting); see Table 5.

Table 4. Single DOF low damping dynamics.

Tool dynamics	**x**	**y**
Natl. freq. (Hz)	750	750
Damping (%)	0.5	0.5
Stiffness (N/m)	5×10^{6}	5×10^{6}
Workpiece dynamics	**x**	**y**
Natl. freq. (Hz)	500	500
Damping (%)	5	5
Stiffness (N/m)	5×10^{10}	5×10^{10}

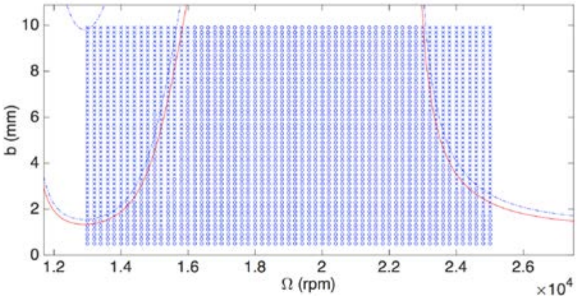


Figure 4. Single DOF low damping case (5% up milling).

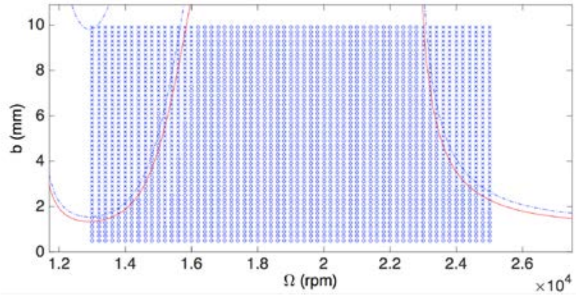


Figure 5. Single DOF low damping case (5% down milling).

Table 5. Single DOF low damping results.

Radial Immersion	Frequency domain analysis	Percent score
5% up milling	Fourier series	93.4
	Average tooth angle	94.8
5% down milling	Fourier series	93.8
	Average tooth angle	95.0

VARIATION OF M METRIC WITH AXIAL DEPTH OF CUT

As defined previously, the M metric defines stability using the difference between successively sampled points. As seen in Fig. 2, the stable (o) and unstable (×) points were identified using a threshold M value of 1 μm (M < 1 μm for stable and M ≥ 1 μm for unstable). This section describes how the metric value changes with an increase in axial depth of cut. The variation in the M values with axial depth of cut and the corresponding change in behavior (stable vs. unstable) is displayed in Fig. 6. Results for multiple spindle speeds are included. It is seen that the M value changes rapidly at the onset of instability.

For example, at a spindle speed of 11500 rpm, the M value for the first unstable point is 7 μm. For 15500 rpm, it is 6 μm. For 22000 rpm, it is 17 μm and it is 26 μm for 25000 rpm. This validates the 1 μm threshold applied in this study.

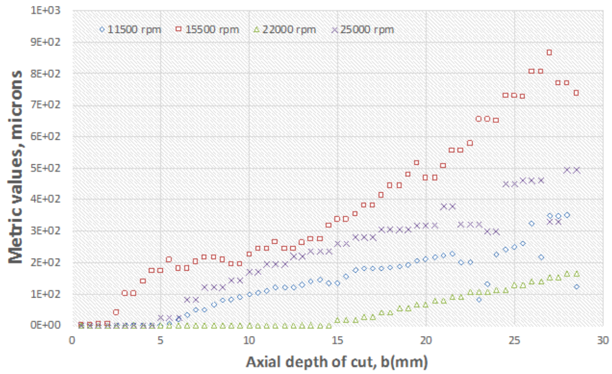


Figure 6. Variation in M with axial depth.

CONCLUSIONS

This paper compared two popular frequency domain milling stability solutions with time domain simulation. A new metric was provided to automatically identify the stability of the time domain simulation results. It was found that the Fourier series analysis more closely matched time domain stability predictions than the average tooth angle approach.

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