BAYESIAN UPDATING USING THE MARKOV CHAIN MONTE CARLO METHOD TO DETERMINE FORCE COEFFICIENTS IN END MILLING

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INTRODUCTION

Machining science, like many other technological fields, enjoyed tremendous gains in the 20th century. It was transformed from predominantly an empirical, trial and error discipline into one that now implements deterministic models for nearly every aspect of machining processes. In milling, for example, models are available to relate stability, part accuracy (from forced vibrations during stable machining), surface finish, and residual stresses to the selected operating parameters, material and tool properties, tool geometry, and part-tool-holder-spindle-machine dynamics. While the 21st century state-of-the-art continues to progress, obstacles remain. For example, all models include uncertainties in their inputs, as well as in the models themselves (due to the underlying assumptions and lack of knowledge). Also, the global marketplace has placed a premium on reduced production time and cost without sacrificing quality. Finally, implementation at the shop floor level can differ substantially from the research laboratory environment due to the availability of pre-process information.

To address these pressing issues, the authors are attempting to formulate milling, which is a critical value-added process in manufacturing, as a decision problem under uncertainty. As part of this work, efforts to implement Bayesian inference to determine the coefficients of a common force model used in end milling performance prediction are described.

MILLING FORCE MODEL

Milling forces can be described using the model provided in Eq. 1. In this equation, F_t is the tangential force component, K_t is the tangential cutting force coefficient, b is the axial depth of cut, h is the instantaneous chip thickness (which depends on the feed per tooth), K_{te} is the tangential edge (plowing) coefficient, F_n is the normal force component, K_n is the normal cutting force coefficient, and K_{ne} is the normal edge coefficient [1].

$$\begin{aligned} F_t &= K_t bh + K_{te} b \\ F_n &= K_n bh + K_{ne} b \end{aligned} \qquad (1)$$

The four coefficients in Eq. 1 are typically obtained by measuring the cutting forces with a dynamometer while machining at known axial depth and feed per tooth values. Measurements are performed over a range of feed per tooth values and a linear regression to the mean *x* (feed) and *y* direction forces is performed.

BAYESIAN INFERENCE

Bayesian inference models form a normative and rational method for updating beliefs when new information is available. Let the prior distribution about an uncertain event, *A*, at a state of information, &, be {*A*|&}, the likelihood of obtaining an experimental result *B* given that event *A* occurred be {*B*|*A*,&}, and the probability of receiving experimental result *B* (without knowing *A* has occurred) be {*B*|&}. Bayes' rule determines the posterior belief about event *A* after observing the experiment result *B*, {*A*|*B*,&} as show in Eq. 2.

$$\{A|B,\&\} = \frac{\{A|\&\}\{B|A,\&\}}{\{B|\&\}} \qquad (2)$$

The product of the prior and likelihood functions is used to determine the posterior distribution. In the case of multiple measurements, the posterior distribution after the first measurement or update becomes the prior for the second and so on.

BAYESIAN UPDATING OF MILLING FORCE COEFFICIENTS

In this paper, Bayesian updating using the Markov Chain Monte Carlo (MCMC) method to determine the force model coefficients in end milling is presented. As noted, Bayesian inference provides a systematic and formal way of updating beliefs when new information is available taking into account the uncertainty in variables. For the case of updating force coefficients using experimental force data, Bayes' rule is written as:

$$f_{K_t,K_n,K_{te},K_{ne}}\left(K_t,K_n,K_{te},K_{ne}\middle|\overline{F}_{x,m},\overline{F}_{y,m}\right)\propto f_{K_t,K_n,K_{te},K_{ne}}\,l\left(\overline{F}_{x,m},\overline{F}_{y,m}\middle|K_t,K_n,K_{te},K_{ne}\right) \quad (3)$$

where $f_{K_t,K_n,K_{te},K_{ne}}\left(K_t,K_n,K_{te},K_{ne}\middle|\overline{F}_{x,m},\overline{F}_{y,m}\right)$ is the posterior distribution of the force coefficients given measured values of the mean forces in the x and y directions, $\overline{F}_{x,m}$ and $\overline{F}_{y,m}$, $f_{K_t,K_n,K_{te},K_{ne}}$ is the prior distributions of the force coefficients, and $l\left(\overline{F}_{x,m},\overline{F}_{y,m}\middle|K_t,K_n,K_{te},K_{ne}\right)$ is the likelihood of obtaining the measured mean force values given specified values of the force coefficients. In this notation, the subscript m denotes measured values from cutting experiments. The measured values were assumed to be statistically independent.

In the case of updating force coefficients as described by Eq. 3, the prior is a joint pdf of the force coefficients, K_t, K_n, K_{te}, and K_{ne}. As a result, the posterior is also a joint pdf of the force coefficients. In Bayesian inference, the MCMC technique can be used to sample from multivariate posterior distributions. The single-component Metropolis Hastings (MH) algorithm facilitates sampling from multivariate distributions without sensitivity to the number of variables [2-7]. To sample from a joint pdf, the algorithm samples one variable at a time and then proceeds sequentially to sample the remaining variables. The joint posterior pdf is the target pdf for MCMC. As noted, the posterior (target) pdf is the product of the prior and likelihood density functions.

EXPERIMENTAL RESULTS

This section describes the experimental setup used to perform force coefficient measurements. Experiments were performed using a 19 mm diameter inserted endmill (one square uncoated Kennametal 107888126 C9 JC carbide insert; zero rake and helix angles, 15 deg relief angle, 9.53 mm square x 3.18 mm). The workpiece material was 1018 steel. The cutting force was measured using a table mounted dynamometer (Kistler 9257B). The first test was completed at a spindle speed, Ω, of 2500 rpm with a 3 mm axial depth of cut and 4.7 mm radial depth of cut (25% radial immersion, RI). The force coefficients were evaluated by performing a linear regression to the mean x (feed) and y direction forces obtained over a range of feed per tooth values: f_t = {0.03, 0.04, 0.05, 0.06, and 0.07} mm/tooth. Figures 1 and 2 show the linear least squares best fit to the experimental mean forces in the x and y directions, respectively. The mean forces show a linear increase for both the x and y directions and the quality of fit is good (R^2 = 0.99). The force coefficients were determined using the slopes and intercepts from the data regression. The values of the experimental mean forces values are provided in Table 1. The force coefficient values calculated using the linear regression were: K_t = 2149 N/mm^2, K_n = 1290.1 N/mm^2, K_{te} = 37.1 N/mm and K_{ne} = 37.1 N/mm.

TABLE 1. Experimental mean forces in x and y directions for 25% radial immersion.

f_t (mm/tooth)	Mean F_x (N)	Mean F_y (N)
0.03	–11.50	40.13
0.04	–13.31	46.10
0.05	–14.83	50.03
0.06	–17.64	56.63
0.07	–19.10	62.06

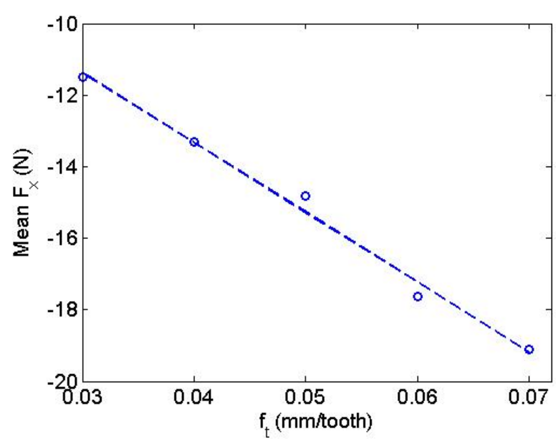

FIGURE 1: Linear regression to the mean forces in the x direction at 25% radial immersion.

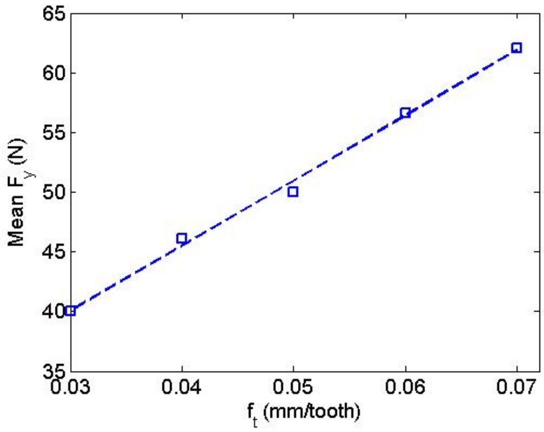

FIGURE 2: Linear regression to the mean forces in the y direction at 25% radial immersion.

The experimental force data listed in Table 1 was used to perform Bayesian updating on the force coefficients using the single component MH MCMC algorithm. The sampling was completed one coefficient at a time in the order $K_t \rightarrow K_n \rightarrow K_{te} \rightarrow K_{ne}$. The single-component MH algorithm facilitates sampling from the joint posterior pdf of the force coefficients, K_t, K_n, K_{te}, and K_{ne}. The posterior joint pdf was the target pdf for the MH algorithm. Table 2 lists the force coefficients obtained using the linear regression and Bayesian updating, where $N(\mu,\sigma)$ identifies the mean, μ, and standard deviation, σ, for the normal distributions.

TABLE 2. Force coefficient values obtained from linear regression and Bayesian updating using mean force values at 25% radial immersion.

	K_t (N/mm^2)	K_n (N/mm^2)	K_{te} (N/mm)	K_{ne} (N/mm)
Least squares	2149.0	1290.1	34.7	37.1
Bayes updating	*N*(2116, 137.3)	*N*(1284, 130.2)	*N*(35.5, 3.2)	*N*(37.4, 3.2)

For this analysis, the prior distribution of force coefficients was assumed to be a joint uniform distribution. Force coefficients were assumed to be independent for the prior. The marginal prior pdfs of the force coefficients were specified as: K_t (N/mm^2) = U(0, 3000), K_n (N/mm^2) = U(0, 3000), K_{te} (N/mm) = U(0, 100), and K_{ne} (N/mm) = U(0, 100), where U represents a uniform distribution and the parenthetical terms indicated the lower and upper values of the range. An uncertainty of 1 N standard deviation was assumed in the mean force data, which was based on the user's belief regarding experimental uncertainty in measured force values. The mean force values were calculated using the current state of the chain for the specified cut geometry [1]. The likelihood for the x and y directions was the value of each pdf for the experimental mean forces.

A second test was completed at 50% RI with all other parameters the same. Figures 3 and 4 show the linear least squares best fit to the experimental mean forces in the x and y directions. The mean force in x direction does not show a clear linear trend (because it is approximately zero for a 50% RI and near the noise limit) and, therefore, the quality of fit is not good (R^2 = 0.70). The least squares fit to the y direction mean forces is very good (R^2 = 0.99), however. The cutting force coefficients, K_t and K_n, and the edge coefficients, K_{te} and K_{ne}, are not decoupled at partial radial immersions but depend on the slopes and intercepts of the least squares fits in both the x and y directions. Therefore, a poor fit in the x direction mean forces affects the values of all coefficients. Table 3 shows the mean forces.

TABLE 3. Experimental mean forces in x and y directions for 50% radial immersion.

f_t (mm/tooth)	Mean F_x (N)	Mean F_y (N)
0.03	1.51	63.35
0.04	1.11	74.71
0.05	0.93	84.98
0.06	0.67	95.29
0.07	-0.54	105.51

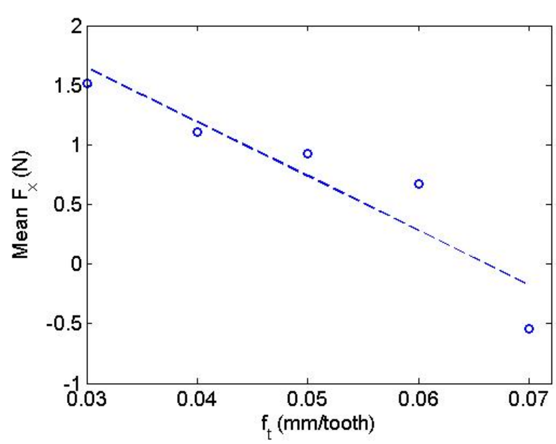

FIGURE 3: Linear regression to the mean forces in the x direction at 50% radial immersion.

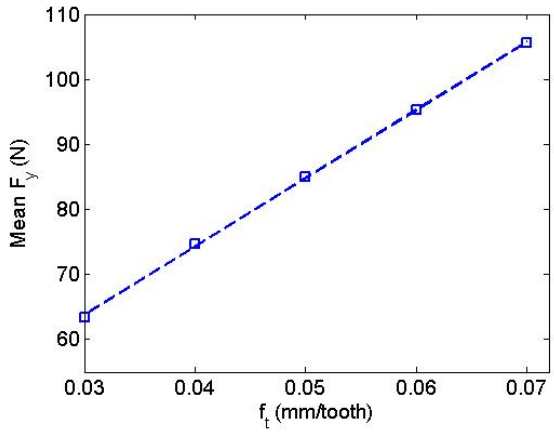

FIGURE 4: Linear regression to the mean forces in the y direction at 50% radial immersion.

The force coefficient values calculated using the linear regression were: K_t = 2504.6 N/mm^2, K_n = 1446.2 N/mm^2, K_{te} = 37.5 N/mm and K_{ne} = 45.2 N/mm. The mean force data listed in Table 3 was also used to update the force coefficients distribution by the MCMC algorithm. Table 4 lists the force coefficients obtained using the linear regression and Bayesian updating. It is seen that the force coefficient values obtained from the two methods do not agree. This is due to the poor least square fit for the mean *x* direction force. Because the Bayesian updating does not rely on a curve fit, the posterior distributions are not affected by the quality of the fit and are insensitive to the radial immersion (as expected); for Bayesian updating, the posterior distributions obtained at 25% and 50% RI agree closely (see Tables 2 and 4).

TABLE 4. Force coefficient values obtained from linear regression and Bayesian updating using mean force values at 50% radial immersion.

	K_t (N/mm^2)	K_n (N/mm^2)	K_{te} (N/mm)	K_{ne} (N/mm)
Least squares	2504.6	1446.2	37.5	45.2
Bayes updating	*N*(2052, 67.8)	*N*(1187, 68.9)	*N*(30.4, 2.3)	*N*(36.7, 2.6)

CONCLUSIONS

Bayesian updating of the force coefficients using the Markov Chain Monte Carlo (MCMC) method was presented. The single component Metropolis Hastings algorithm of MCMC was used. Bayesian inference provides a formal way of belief updating when new experimental data is available. Bayesian updating gives a posterior distribution that incorporates the uncertainty in variables as compared to traditional methods like the linear regression which give a deterministic value. By combining prior knowledge and experimental results, Bayesian inference reduces the number of experiments required for uncertainty quantification. Using Bayesian updating, a single test can give a distribution for force coefficients. The posterior distribution samples provide the covariance of the joint distribution as well. Experimental milling results showed that the linear regression approach did not give consistent results at 50% RI due to a poor quality of fit in the *x* direction mean forces, whereas Bayesian updating yielded consistent results at both radial immersions tested. Also, since Bayesian updating does not rely on a least squares fit, mean force data at different feed per tooth values is not required.

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