

IMPROVEMENT OF A PERIODIC ERROR COMPENSATION ALGORITHM BASED ON THE CONTINUOUS WAVELET TRANSFORM

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INTRODUCTION

Heterodyne displacement measuring interferometry provides important metrology for applications requiring high resolution and accuracy, such as in the semiconductor manufacturing industry and for linear stage calibration. Heterodyne Michelson interferometers use a two-frequency laser source and separate the two optical frequencies into one fixed length and one variable length path via polarization. Ideally these two beams are linearly polarized and orthogonal so that only one frequency is directed toward each path. An interference signal is obtained by recombining the light from the two paths; this results in a measurement signal at the heterodyne (split) frequency of the laser source. This measurement signal is compared to the optical reference signal. Motion in the measurement arm causes a Doppler shift of the heterodyne frequency which is measured as a continuous phase shift that is proportional to displacement. In practice, due to misalignment of optical components, component imperfections, and elliptical polarization, undesirable frequency mixing occurs which yields periodic errors [1-3]. Typically, 1st, 2nd and even higher order periodic errors occur, which correspond to the number of periods per fringe displaced, as shown in Figure 1. A displacement fringe corresponds to the wavelength divided by the interferometer fold factor which is determined by the interferometer setup. Ultimately, this error can limit the accuracy to approximately the nanometer level.

Many studies have investigated the measurement and compensation of periodic error, including frequency domain [4] and time domain approaches [5, 6]. For frequency domain approach, the periodic error can be measured by calculating the Fourier transform of the time domain data collected during constant velocity

target displacement. However, this method is not well suited to non-constant velocity profile and sub-fringe positioning ranges. An alternate digital algorithm which can be applied in real-time for constant or non-constant velocity motions is also available for measuring and compensating 1st and 2nd order periodic error. But this method leaves 3rd and higher order periodic errors as residual errors.

In this research, a real-time continuous wavelet transform (CWT) based algorithm, which was developed in previous work [7], is improved and used to compensate 1st, 2nd and higher order periodic errors (modeled as pure sine signals). Moreover this algorithm can compensate errors in both constant and non-constant velocity motions.

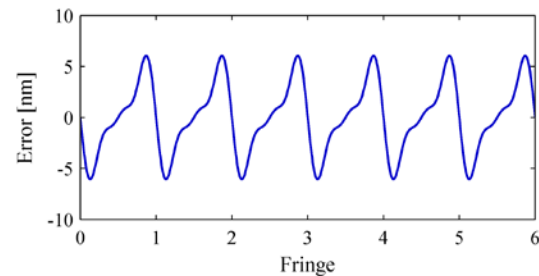


FIGURE 1. Example of 1st, 2nd, and 3rd order periodic error as a function of fringes. Typically, 1st order error has a larger magnitude than higher order errors.

CONTINUOUS WAVELET TRANSFORM

The wavelet transform can be used to analyze time series data that contains non-stationary (variable period) power at multiple frequencies [8]. Wavelet functions refer to either orthogonal or non-orthogonal wavelets. The choice of the appropriate wavelet transform (continuous or discrete) and wavelet function is based on

whether the purpose of data analysis is detection or compression [9].

A wavelet function $\psi(t)$ is a finite energy function [10] with an average of zero,

$$\int_{-\infty}^{+\infty} \psi(t) dt = 0. \quad (1)$$

A wavelet family is generated by dilating the mother wavelet via the scale $s > 0$ and translating it via the location $u \in \mathfrak{R}$. This series of wavelets can be expressed as

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right). \quad (2)$$

In this research, a continuous wavelet transform (CWT) is used to analyze the signal $x(t)$, with a wavelet function $\psi(t)$. For a one-dimensional signal $x(t)$, the CWT is defined as the convolution of $x(t)$ with a scaled and translated version of $\psi(t)$ via

$$\begin{aligned} Wx(u, s) &= \int_{-\infty}^{+\infty} x(t) \psi_{u,s}^*(t) dt \\ &= \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi^*\left(\frac{t-u}{s}\right) dt \end{aligned} \quad (3)$$

where $\psi^*(t)$ is the wavelet function, s is the scale, and u is the location. In the present work, the complex Morlet wavelet is used as the mother wavelet

$$\psi^*\left(\frac{t-u}{s}\right) = \pi^{-\frac{1}{4}} e^{i2\pi f_0 \frac{t-u}{s}} e^{-\frac{1}{2}\left(\frac{t-u}{s}\right)^2}. \quad (4)$$

In practice, Equation 3 must be converted from continuous to discrete:

$$Wx(n, s) = \sum_{n'=1}^M \left(x(n') \sqrt{s} \psi^*\left(\frac{(n'-n)\Delta t}{s}\right) \Delta t \right), \quad (5)$$

where $x(n)$ is the n^{th} discrete data point, ψ^* is the mother wavelet, M is the number of total data points in the signal, and Δt is the sampling time.

After applying the complex Morlet wavelet to the signal, the CWT result is a two-dimensional complex array. This array can be used to extract the CWT ‘‘ridge’’ and, therefore, the phase of the periodic errors. The ridge is the location where the CWT coefficient reaches its local maximum

along the scale direction [11]; the coefficient is maximum when the analysis frequency equals the signal frequency [12]. The ridge and phase are

$$\text{ridge}(n) = \max \{ |Wx(n, s)| \} \quad \text{and} \quad (6)$$

$$\phi(n, s) = \arctan \left(\frac{\text{Im}(Wx(n, s_{\text{ridge}}))}{\text{Re}(Wx(n, s_{\text{ridge}}))} \right), \quad (7)$$

where s_{ridge} is the scale at the ridge, and Im and Re represent the imaginary and real parts of the CWT coefficient, respectively.

COMPENSATION ALGORITHM

The periodic error compensation, which can be processed in real-time, is depicted in Figure 2.

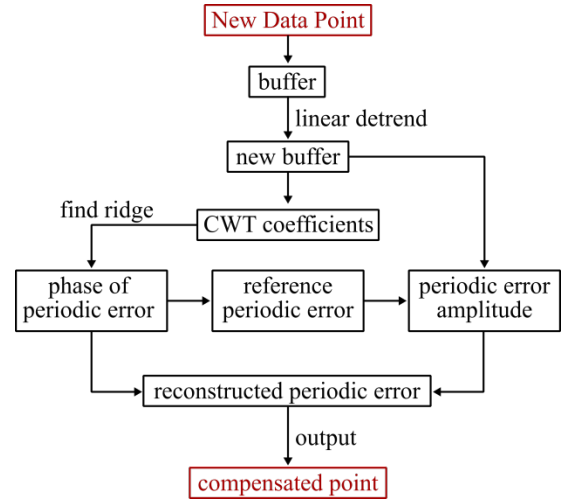


FIGURE 2. Calculations to implement the periodic error compensation algorithm.

Each time a new data point is obtained, an N -size array is populated with the last N data points. With this series of data, the CWT is computed at the last data point (i.e., location n in Equation 5 is fixed at the end of the array). Therefore, an array of coefficients at the end point along scales is obtained and the ridge can be determined at scale s_1 . This scale corresponds to the 1st order periodic error frequency. Because the scale is inversely related to the frequency, the scale $s_i = s_1 / i$ corresponds to the i^{th} order periodic error frequency.

For each new data point, the ridge and phase is calculated, so the periodic error phase

information is determined. Arrays for the reference j^{th} order periodic error are constructed,

$$r_j[1\dots n] = \begin{cases} \sin(j\varphi(1)), \sin(j\varphi(2)), \dots, \\ \sin(j\varphi(N)) \end{cases}. \quad (8)$$

We consider a general form of m order periodic errors,

$$\sum_{j=1}^m A_j r_j[1\dots n], \quad (9)$$

where A_j is j^{th} order periodic error amplitude. Apply CWT linearity property to obtain equations:

$$\begin{cases} c_1 = A_1 d_{11} + A_2 d_{12} + \dots + A_m d_{1m} \\ c_2 = A_1 d_{21} + A_2 d_{22} + \dots + A_m d_{2m} \\ \vdots \\ c_m = A_1 d_{m1} + A_2 d_{m2} + \dots + A_m d_{mm} \end{cases}, \quad (10)$$

where c_i is the CWT result for the data array at scale s_i , and d_{ij} is the CWT result for reference j^{th} order periodic error at scale s_i .

The amplitudes can be solved and then the periodic error is reconstructed as

$$\sum_{i=1}^m A_i \sin(i\varphi(N)).$$

Finally, this result is subtracted from the original data to determine the compensated displacement data point.

SIMULATIONS

Simulations were used to assess the validity of the CWT-based algorithm.

The algorithm was applied to a simulated constant velocity motion (50 mm/min) with 1st, 2nd, and 3rd order periodic error amplitudes of 4 nm, 2.5 nm, and 1 nm, respectively. The measured amplitudes are shown in Figure 3. The relative errors are 3.1%, 3.8%, and 8.9%, respectively. The compensated result is displayed in Figure 4. The root-mean-square (RMS) error is decreased by approximately 86.4%.

The algorithm was also verified using a simulated accelerating motion (30 mm/min² acceleration, 4 nm, 2.5 nm, and 1 nm amplitudes for 1st, 2nd, and 3rd order periodic errors, respectively). The measured amplitudes are shown in Figure 5. The relative errors are 8.6%, 4.8%, and 10.8%, respectively. The compensated result is provided in Figure 6. The RMS error is reduced by approximately 80.1%.

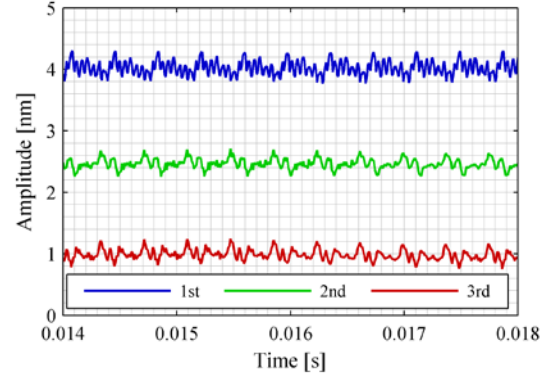


FIGURE 3. The measured amplitudes of 1st, 2nd, and 3rd order periodic errors in the constant velocity motion.

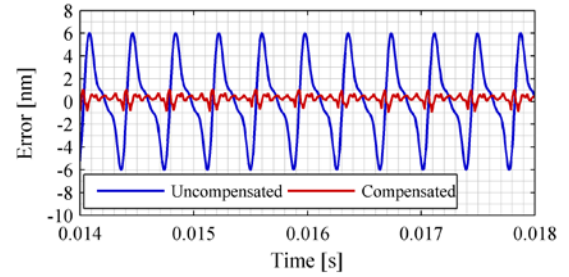


FIGURE 4. The result of periodic error compensation in the constant velocity motion.

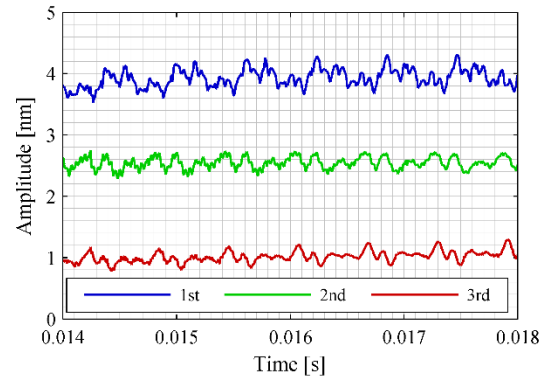


FIGURE 5. The measured amplitudes of 1st, 2nd, and 3rd order periodic errors in the non-constant velocity motion.

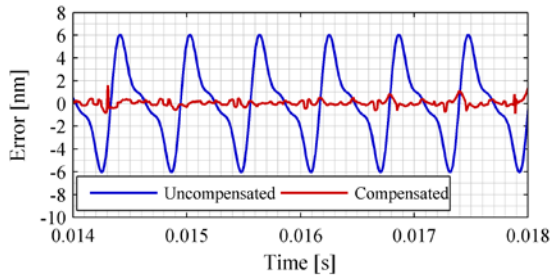


FIGURE 6. The result of periodic error compensation in the non-constant velocity motion.

CONCLUSIONS

A CWT-based algorithm is improved and used in this research to compensate higher order periodic error in heterodyne interferometer displacement signals for constant and non-constant velocity motions. Future work will focus on implementing this algorithm on the hardware and compensating the experiment-collected data in real-time.

ACKNOWLEDGEMENT

This work was supported by the National Science Foundation CMMI division under award collaborative research awards 1265881, 1265842, and 1265824.

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