

**REMAINING USEFUL TOOL LIFE PREDICTIONS USING BAYESIAN INFERENCE**

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**ABSTRACT**

Tool wear is an important limitation to machining productivity. In this paper, remaining useful tool life predictions using the random walk method of Bayesian inference is demonstrated. End milling tests were performed on a titanium workpiece and spindle power was recorded. The power root mean square value in the time domain was found to be sensitive to tool wear and was used for tool life predictions. Sample power root mean square growth curves were generated and the probability of each curve being the true growth curve was updated using Bayes' rule. The updated probabilities were used to determine the remaining useful tool life. Results show good agreement between the predicted tool life and the true remaining life. The proposed method takes into account the uncertainty in tool life and the percentage of nominal power root mean square value at the end of tool life.

**INTRODUCTION**

Manufacturing processes are rarely optimal because conservative operating parameters are selected to allow for process variations during production. This can occur in machining applications with hard-to-machine materials, such as titanium, for example. One parameter that can result in efficiency loss is cutting tool life estimation. Tool wear is considered to be a stochastic process and, therefore, difficult to predict. In this work, the limitation to machining productivity imposed by tool wear is addressed using Bayesian inference techniques. A new approach that establishes an estimate of the remaining useful life (RUL) for a selected tool based on the time domain root mean square (RMS) spindle power during machining is described.

A tool condition monitoring system may be used to determine when a tool change is necessary; implementation is composed of three steps: 1) identify and extract relevant features correlated to tool wear; 2) train the system using tool wear experiments; and 3) develop an intelligent inference technique for predicting tool wear [1,2]. Tool condition monitoring systems which rely on mathematical models generally requires a significant amount of empirical data and, therefore, are challenging to apply in industrial applications [1]. Another important limitation to tool condition monitoring is the stochastic nature of the sensor signal due to large-scale variation and non-homogeneities in the workpiece [3]. Therefore, a tool condition monitoring system which relies on a deterministic threshold value may not be reliable due to the uncertainty in the threshold value at the end of tool life. The proposed method takes into account the inherent uncertainty in tool life and the threshold value of the sensor signals at the end of tool life. Research has shown acoustic emission, vibration signals, cutting forces (static and dynamic), and spindle power/current to be sensitive to tool wear; a review of the sensor signals is presented in [1]. In this work, spindle power is used for tool condition monitoring since it is non-intrusive and is sensitive to tool wear [4-5]. Furthermore, it is a convenient industrial measurement because it offers a lower cost alternative to the measurement of most other relevant physical quantities. Although Bayesian methods have gained popularity in recent years, its application to tool condition monitoring has been limited in the literature. Elangovan *et al.* used a combination of Naïve Bayes and Bayes' network for tool condition monitoring in a discrete case [6]. Dey *et al.* used Bayesian networks for root cause analysis of process variations [7]. Karandikar *et al.* used Bayesian inference for tool life

predictions in pre-process planning stage [8]. The main contribution of this paper is to demonstrate and validate a novel random walk method of Bayesian inference for remaining useful tool life predictions.

The remainder of the paper is organized as follows. First, an overview of Bayesian inference is provided. Second, training experiments are described that identify the influence of tool wear on spindle power and the corresponding tool life. The training experiment data was used to determine the prior. Third, the random walk method for Bayesian updating is described. Fourth, two additional experiments are detailed and the remaining useful tool life is predicted using the measured power RMS data. Finally, remaining useful tool life predictions are compared to the true remaining life.

### BAYESIAN INFERENCE

As noted, tool life is considered stochastic and, in general, difficult to predict. This is due to many factors such as the complex nature of the tool wear phenomenon and tool-to-tool performance variation. Therefore, tool life should be characterized by a probability distribution to incorporate its inherent uncertainty.

Bayesian inference models form a normative and rational method for updating beliefs when new information is available. A Bayesian model treats an uncertain quantity as a random variable using a probability distribution. Let the prior distribution about an uncertain event,  $A$ , be  $P(A)$ , the likelihood of obtaining an experimental result  $B$  given that event  $A$  occurred be  $P(B|A)$ , and the probability of receiving experimental result  $B$  (without knowing  $A$  has occurred) be  $P(B)$ . Bayes' rule is used to determine the posterior belief about event  $A$  after observing the experiment results,  $P(A|B)$ , as shown in Eq. 1.

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} \quad (1)$$

The product of the prior and likelihood function is used to calculate the posterior distribution. For multiple measurements, the posterior distribution after the first measurement, or update, becomes the prior for the second, and so on. An important requirement for applying Bayes' rule in this case is selecting the initial belief (prior distribution) for the tool life. In general, this initial prediction: 1) can be constructed from any combination of theoretical considerations, previous experimental results, and expert opinions; and 2) should be chosen to be as informative as possible using the experimenter's belief.

### TRAINING EXPERIMENT

The experimental steps followed to collect the tool wear data for a 10 mm diameter endmill (Sandvik Coromill 316-10SM450-10010P body with a 10-A16-SS-075 solid carbide fluted insert) are described in this section. The workpiece material was Ti 6Al-4V. The down milling tests were completed at a spindle speed,  $\Omega$ , of 3660 rpm (cutting speed = 115 m/min

with a 2 mm axial depth of cut and 2.5 mm radial depth of cut (25% radial immersion). A through-tool flood coolant at 50 bar pressure was used. The feed per tooth value was 0.06 mm/tooth. The spindle power was monitored during cutting using a VYDAS power transducer connected to a National Instruments cDaq data acquisition module. The RMS power,  $p_{rms}$ , in the time domain was sampled over a 6 second cutting interval once for each pass (every 12.5 seconds). The insert wear profile was measured every six passes. Tool life was defined as the time required for the maximum flank wear width,  $FWW$ , to reach 0.3 mm. No crater wear was observed. Calibrated digital images were used to identify the  $FWW$ . The first measurement was performed after the first pass to determine the initial rapid rise in  $FWW$ . Figure 1 shows the experimental setup. Figure 2 shows the growth of  $FWW$  with machining time for all the teeth. Three stages of  $FWW$  growth were observed such as rapid initial wear, uniform wear and final wear [9]. As shown in Figure 2, the  $FWW$  was found to be consistent for all four teeth. The tool life was found to be 22.0 minutes on the first tool tested. Figures 3 and 4 show the growth in  $p_{rms}$  with machining time and average  $FWW$ , respectively. As shown in Figure 4,  $p_{rms}$  shows a clear correlation to  $FWW$ . Note that in Figure 3 and Figure 4, the  $p_{rms}$  values are denoted in volts as given by the VYDAS power transducer connected to an NI cDaq device. The sensor  $p_{rms}$  values (reported in V) were normalized by the initial value such that all subsequent sensor values were shown as a percentage of the nominal value. Note that the nominal value is the initial value of  $p_{rms}$  measured at the first pass. The purpose for this method is to enable the same scale to be used across all cutting processes and machining parameters. This method is also used in commercial systems such as ARTIS. Figure 5 shows the normalized  $p_{rms}$  values, expressed as percent of the nominal, from the experiment.

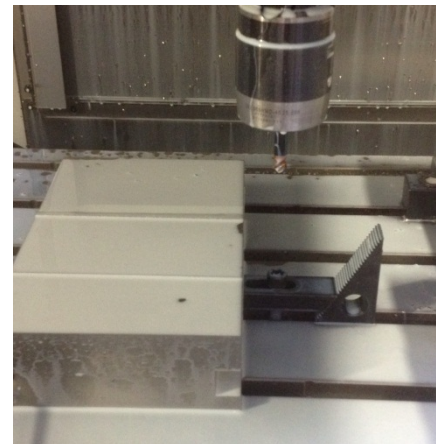


Figure 1. Experimental setup.

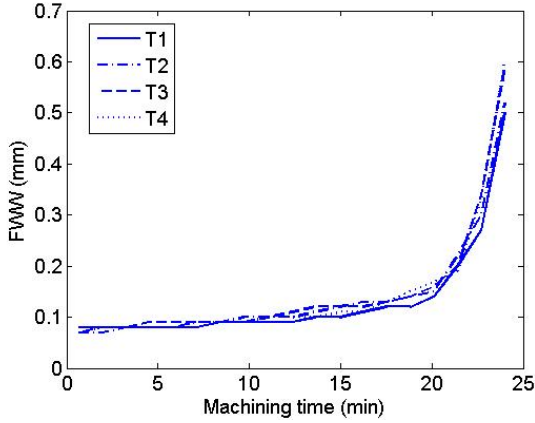


Figure 2. Growth in  $FWW$  with machining time (four teeth).

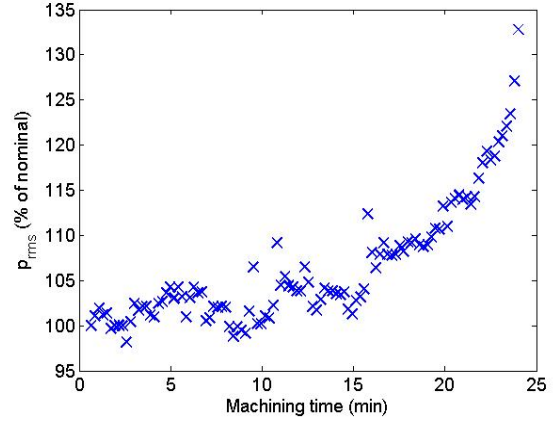


Figure 5.  $p_{rms}$  expressed as percent of the nominal value.

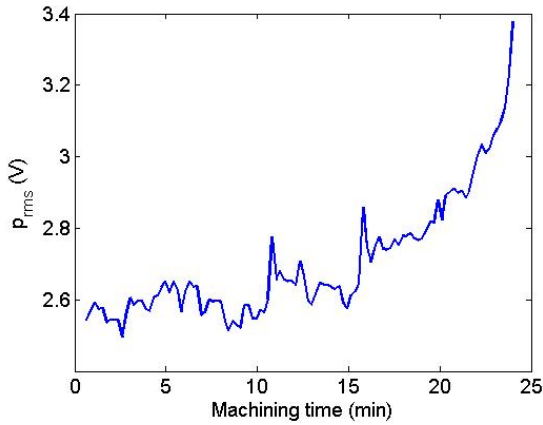


Figure 3. Growth in  $p_{rms}$  with machining time.

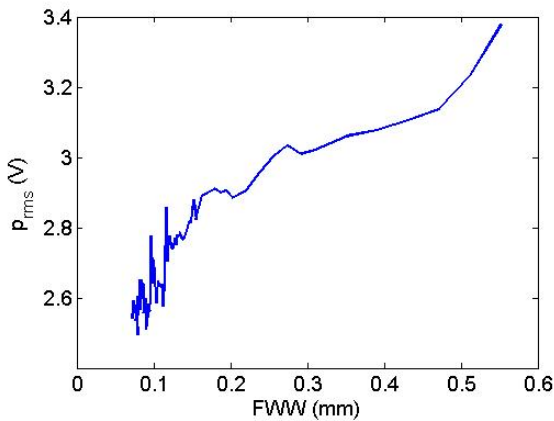


Figure 4. Growth in  $p_{rms}$  with average  $FWW$ .

### BAYESIAN INFERENCE USING RANDOM WALK METHOD

Bayesian inference provides a rigorous mathematical framework for belief updating about an unknown variable when new information becomes available. In this study, the tool RUL was estimated using a random walk method for Bayesian updating. As shown in Figures 3 and 4,  $p_{rms}$  is sensitive to tool wear and was therefore used to perform the Bayesian updating. Although a single sensor was used in this study, the method can be extended to include multiple sensor outputs. The  $p_{rms}$  increases with  $FWW$  and, therefore, machining time. The growth in  $p_{rms}$  was approximated using a second order least squares fit ( $R^2 = 0.87$ ). The random walk method for Bayesian updating proceeds by generating sample  $p_{rms}$  growth curves, each of which represents the true  $p_{rms}$  growth curve with some probability. These sample paths were used as the prior for Bayesian inference. In this work, it was (initially) assumed that each sample curve is equally likely to be the true curve. This implies that if  $N$  sample  $p_{rms}$  growth curves, or sample paths, are generated, each represents the true  $p_{rms}$  growth curve with an equal prior probability of  $1/N$ . The prior probability of the sample paths was updated by applying Bayes' rule to  $p_{rms}$  measurements. For each sample path, Bayes' rule can be written as the following product:

$$P(\text{path} = \text{true } p_{rms} \text{ growth curve} | \text{test result}) \propto P(\text{test result} | \text{path} = \text{true } p_{rms} \text{ growth curve}) \times P(\text{path} = p_{rms} \text{ growth curve})$$

where  $P(\text{path} = p_{rms} \text{ growth curve})$  is the prior probability that a given path is the true  $p_{rms}$  growth curve. As noted, the probability is assumed to be  $1/N$  before any  $p_{rms}$  measurement is completed since each path is considered equally likely to be the true  $p_{rms}$  growth curve. Also,

$$P(\text{test result}|\text{path} = \text{true } p_{rms} \text{ growth curve})$$

is referred to as the likelihood and

$$P(\text{path} = \text{true } p_{rms} \text{ growth curve}|\text{test result})$$

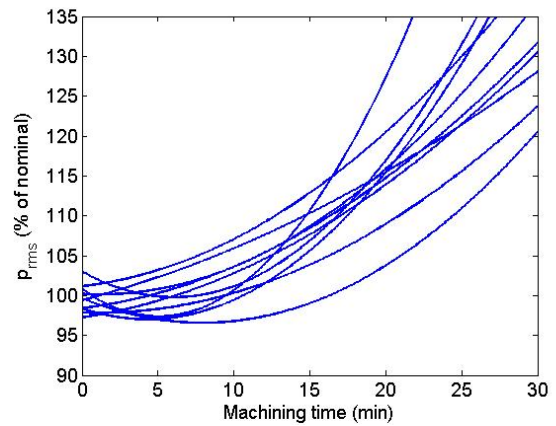
is the posterior probability of the sample  $p_{rms}$  growth curve given the test result. The product is normalized so that the sum is equal to one.

## ESTABLISHING THE PRIOR

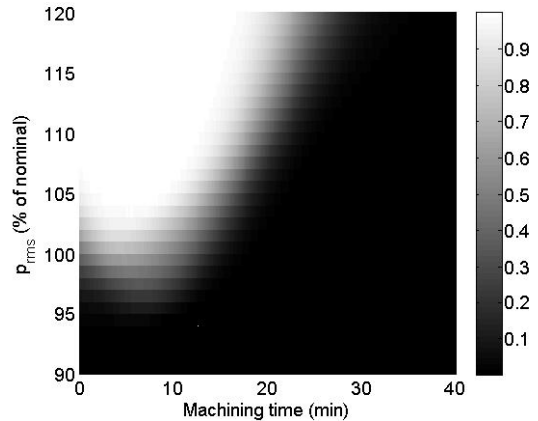
There is uncertainty in the growth of  $p_{rms}$  as a function of machining time and the tool life. It was approximated using a second order polynomial. The prior incorporated uncertainty (standard deviation) in the measurement due to noise and uncertainty in the second order assumption. The combined uncertainty level was set at 3% (of the nominal value). A normal distribution was assumed with mean of 100% and standard deviation of 3%,  $N(100,3)$  where  $N$  denotes a normal distribution and the numbers in the parenthesis identify the mean and standard deviation, respectively. Additionally, the  $p_{rms}$  value at the end of tool life,  $p_r$ , was assumed to be between 110% and 120% with equal probability,  $U(110, 120)$ , where  $U$  denotes a uniform distribution and the numbers in the parenthesis give the minimum and maximum values, respectively. The tool life was assumed to follow a normal distribution with a mean equal to 22 minutes and standard deviation equal to 3 minutes,  $N(22,3)$ , denoted by  $t_{life}$ . Third, the  $p_{rms}$  value at the end of the initial wear stage, defined by a maximum  $FWW$  of 0.12 mm, was assumed to be between 95% and 105%,  $U(95, 105)$ . The time at the end of the initial wear stage was assumed to be uniformly distributed between 0.5 and 0.75 times the tool life value, denoted by  $t_{life} \times U(0.5, 0.75)$ . The initial wear time and the  $p_{rms}$  values corresponding to the initial wear can be determined by the user using the data shown in Figure 2 and Figure 5. The distributions were based on the training data set shown in Figure 5.

The sample paths were generated as follows. First, random samples were drawn for the initial  $p_{rms}$  value, the  $p_{rms}$  value at the end of initial wear value, and the  $p_{rms}$  value at the end of tool life from the prior distributions. Next, random samples were drawn for the time at the end of initial wear stage and end of tool life from the prior distributions. The time for the initial  $p_{rms}$  was 0 minutes. A second order least squares line was fit to the three values of  $p_{rms}$  and time. The procedure was repeated for  $1 \times 10^4$  sample curves. An alternate method for generating the sample paths is to specify distributions for the coefficients of the second order fit coefficients, for example. Figure 6 shows 10 sample curves. Figure 7 shows the prior cumulative distribution function (cdf) for  $p_{rms}$  as a function of machining time using the second order model. The prior cdf gives the probability,  $P(p_{rms})$ , that  $p_{rms}$  will be less than the selected value as a function of machining time. This is the gray scale value in Figure 7. To illustrate, Figure 8 shows the cdf of  $p_{rms}$  at 15 minutes. From Figure 8, the probability that  $p_{rms}$  is less than 120 after 15 minutes is 1, whereas the probability that  $p_{rms}$  is

less than 105 after 15 minutes is 0.68. The cdf shown in Figure 8 is calculated as follows.  $0.32 \times 10^4$  sample  $p_{rms}$  growth curves out of  $10^4$  sample paths give  $p_{rms} > 105$  at 15 minutes. Therefore,  $P(p_{rms} < 105) = 0.68$  at 15 minutes. Similarly, zero sample  $p_{rms}$  growth curves out of  $10^4$  give  $p_{rms} > 120$  at 15 minutes. Therefore,  $P(p_{rms} < 120) = 0$  at 15 minutes. However, note that this is true only for the prior, where each sample  $p_{rms}$  growth curves is assumed to be equally likely to be the true path (probability equal to  $1/N$ ). The probability of each sample  $p_{rms}$  growth curves being the true curve is updated using experimental results, as shown in subsequent sections. For posterior cdf, the updated probability of each path has to be taken into consideration while calculating the cdf.



**Figure 6. 10 sample curves to model  $p_{rms}$  growth with machining time.**



**Figure 7. Prior cdf of  $p_{rms}$  using the second order model. The color bar denotes the probability that  $p_{rms}$  will be less than the selected value.**

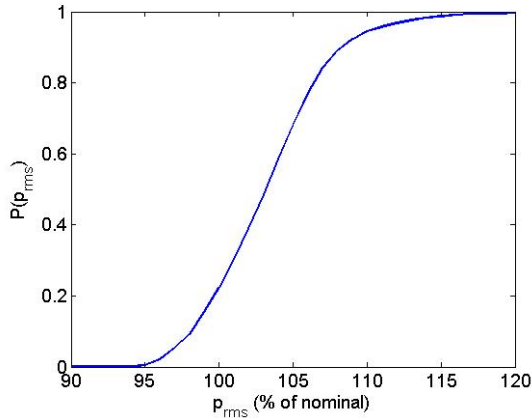


Figure 8. Prior cdf of  $p_{rms}$  at 15 minutes.

### LIKELIHOOD

The probability of the sample  $p_{rms}$  growth curves was updated using  $p_{rms}$  measurements and Bayes' rule. The likelihood function incorporates the uncertainty in the  $p_{rms}$  measurement and the assumed second order model. A non-normalized Gaussian distribution was used as the likelihood in this study shown in Eq. 2:

$$l = e^{\frac{-(p-p_m)^2}{k}}, \quad (2)$$

where  $l$  is the likelihood value,  $p_m$  is the measured  $p_{rms}$ ,  $p$  is the  $p_{rms}$  value for a sample curve at the experimental spindle speed, and  $k$  depends on the tool wear uncertainty. Because the likelihood function is expressed as a non-normalized normal distribution,  $k = 2\sigma^2$ , where  $\sigma$  is the standard deviation of  $p_{rms}$ ; it represents the uncertainty in the  $p_{rms}$  measurement and the assumed second order model. The likelihood function describes how likely it is that the given the  $p_{rms}$  measurement result is obtained at a particular machining time, given that the sample  $p_{rms}$  growth curve is the true curve. If the  $p_{rms}$  growth curve value is near the measurement result, then the likelihood value is high. Otherwise, it is low. The likelihood function can be interpreted as assigning weights from 0 to 1 to the sample curves; 0 implying not likely at all and 1 implying most likely. An increased uncertainty (higher  $\sigma$ ) widens the likelihood function so that comparatively higher weights are assigned to sample curves far from the experimental result. Subsequently, larger uncertainty yields a more conservative estimate of tool life.

### BAYESIAN UPDATING

According to the Bayes' rule, the posterior probability is the product of the prior and the likelihood and normalized so that the sum is equal to unity. The prior probability for each path is  $1 \times 10^{-4}$  and the likelihood value is determined using Eq. 2; the value of  $\sigma$  was taken as 3%. The updated probabilities of the sample paths, calculated as the product of the prior and

likelihood and normalized such that the sum is equal to unity, were used to determine the posterior  $p_{rms}$  cdf. To illustrate, consider a normalized measurement of  $p_m = 105\%$  at 10 minutes. Figure 9 shows the likelihood function for this result with of  $\sigma = 3\%$ . Figure 10 shows the updated cdf of the  $p_{rms}$  curves given  $p_m = 105\%$ . Note that the updated probabilities of the sample growth curves were used to calculate the posterior cdf. There is uncertainty in the value of the  $p_{rms}$  at the end of tool life. Recall that it was assumed that the  $p_{rms}$  value at the end of tool life was equally likely to be between 110% and 120%. The cdf values at a selected  $p_{rms}$  values give the probability of  $FWW$  exceeding the critical  $FWW$  (0.3 mm), which is defined as the end of tool life. The probability that the tool has exceeded the critical  $FWW$  is denoted by  $p_f$ . Each  $p_{rms}$  value has a probability distribution of  $FWW$  being less than the critical  $FWW$ . To illustrate, Figure 11 shows the probability of  $FWW$  being less than critical  $FWW$  at  $pt = 110\%$  and  $pt = 120\%$ . Recall that  $p_t$  denotes the  $p_{rms}$  value at the end of tool life. The 95% RUL implies that there is a 0.05 probability of  $FWW$  exceeding the critical  $FWW$ . Each  $p_t$  value will have a different 95% value which will increase with the value selected.

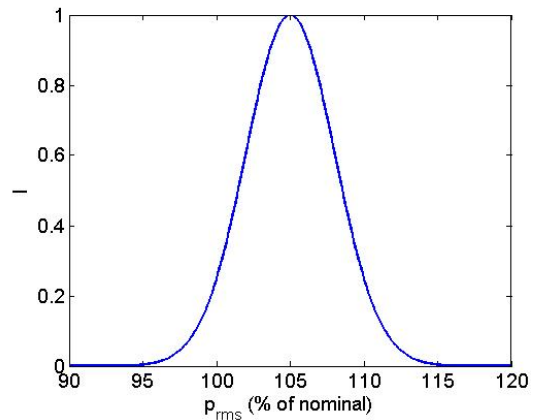


Figure 9. Likelihood function for  $p_m = 105\%$  and  $\sigma = 3\%$ .

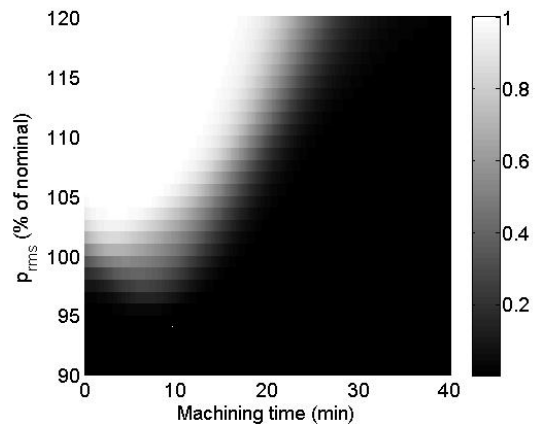
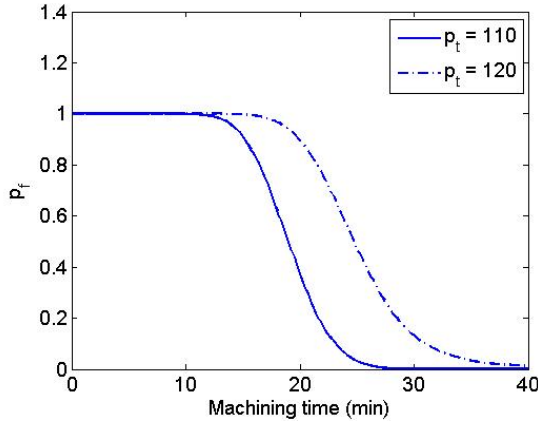


Figure 10. Posterior cdf of  $p_{rms}$  given  $p_m = 105\%$  at 15 minutes.

From Figure 11, the 95% tool life value at  $p_t = 110\%$  is 14.1 minutes; it is 18.6 minutes at  $p_t = 120\%$ . Note that the measurement is completed at 15 minutes; therefore the 95% RUL values were -0.9 minutes and 3.6 minutes at  $p_t = 110\%$  and  $p_t = 120\%$ , respectively. Since it was assumed that it is equally likely that the  $p_{rms}$  value is between 110% and 120% when the tool fails, the expected RUL was calculated as the average value from all the threshold values. For example, if only 110% and 120% values were considered, the 95% RUL was 1.35 minutes.



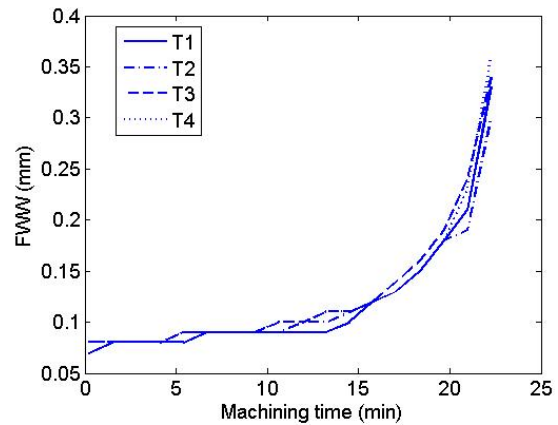
**Figure 11. Probability of FWW being less than critical FWW at  $p_{rms} = 110\%$  and  $p_{rms} = 120\%$ .**

## RUL PREDICTIONS

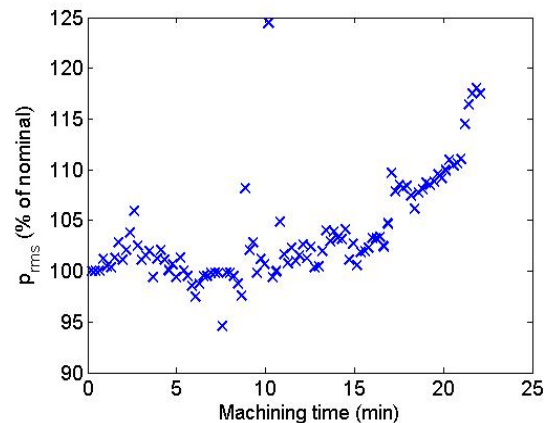
Two tool wear tests were performed at 3660 rpm for RUL predictions. The procedure of the tests was the same as described in the previous section and the other operating parameters were the same as for the training experiment. The FWW was measured after every 6 passes. Figure 12 shows the growth of FWW with machining time for the first test. The tool life was 21.7 minutes. The  $p_{rms}$  value was measured (and normalized as a percentage of the nominal value) after every pass and then used to update the prior probabilities of the sample  $p_{rms}$  growth curves using the procedure described previously. Figure 13 shows the  $p_{rms}$  measured values. The value of  $\sigma$  was assumed to be 3%. Note that the updated probabilities of sample paths were used to calculate the posterior cdf of  $p_{rms}$ . From the updated probabilities of the sample  $p_{rms}$  growth curves, the RUL of the tool before the FWW reaches the critical value was calculated. As noted, a 95% RUL estimate implies that there is a 0.05 probability, of the tool exceeding the critical FWW limit of 0.3 mm. Each  $p_{rms}$  measurement updates the RUL estimates. For the predictions, the  $p_t$  values were discretized into 11 levels from 110% to 120% and the RUL estimate was taken as the average value. Figure 14 shows the posterior cdf of  $p_{rms}$ . Figure 15 shows the 95%, RUL of the tool after each measurement and the true remaining life calculated from the observed tool life value (21.7 minutes). 95% RUL predicts a tool life of 0.2 minutes at the

end of tool life. The initial 95% RUL prediction is conservative due to the uncertainty in the  $p_{rms}$  second order growth model in the prior distribution. The estimate approaches the true life after updating using experimental results.

Note that there is an outlier  $p_{rms}$  value at 10 minutes equal to 125%. This might be due to the material non-homogeneities in the workpiece material or higher than commanded axial depth of cut. A deterministic threshold will cause an erroneous alarm for a worn tool due to the outlier measurement. The proposed method takes into account the uncertainty in the threshold value and the tool life. From Figure 15, it is seen that the RUL estimate drops from 6.0 minutes to 4.5 minutes due to the measured value of 125%. However, subsequent values in the range of 100%-105% cause an increase in RUL estimate to 6 minutes. Thus, the method is robust and insensitive to outlier measurements as it takes into account uncertainties in measurement and tool life.



**Figure 12. Growth in FWW with machining time (four teeth).**



**Figure 13.  $p_{rms}$  measured values (normalized).**

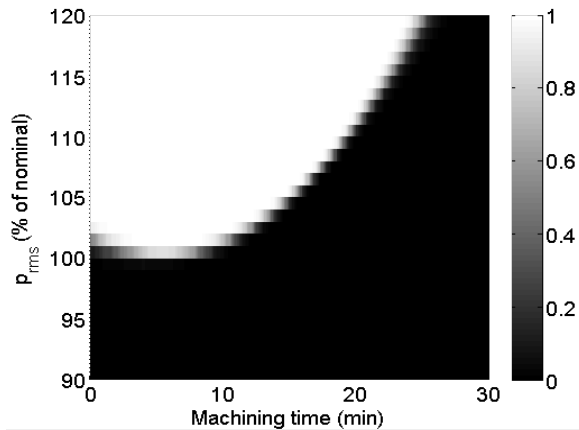


Figure 14. Posterior cdf using  $p_m$  values.

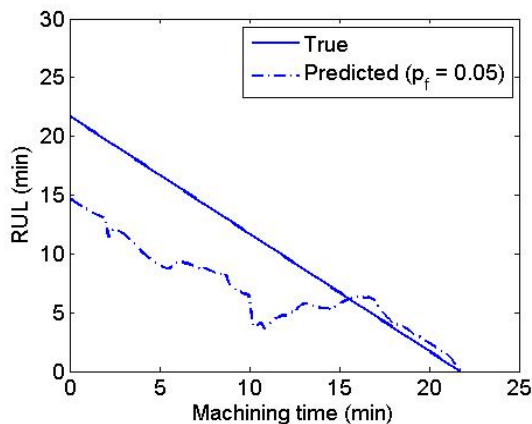


Figure 15. RUL predictions.

Additional testing was completed using the same procedure as described. Figure 16 shows the growth in  $FWW$  and Figure 17 shows the measured (and normalized)  $p_{rms}$  as a function of machining time. The tool life obtained was 17.7 minutes. Again, the  $p_{rms}$  value was measured after every pass, normalized, and then used to update the prior probabilities of the sample  $p_{rms}$  growth curves and estimate the 95% RUL. Figure 18 shows the posterior cdf  $p_{rms}$ . Figure 19 shows RUL predictions for the second test. As shown in Figure 19, the RUL predictions estimate a life of 4.8 minutes at the end of tool life. This is due to end of tool life at a threshold value of 108%. Recall that it was assumed that it is equally likely for the  $p_{rms}$  value to be between 110% and 120% at the end of tool life. Since, the end of tool life occurs at 108%, the method over estimates the remaining life. The advantage of the method is that the prior estimates on the threshold values and tool life can be continuously updated based on observed experimental evidence. For example, for the next test the  $p_{rms}$  value can assumed to be equally likely between 108% and 120% at the end of tool life. To illustrate, the RUL predictions were repeated assuming that  $p_{rms}$  value can assumed to be equally likely between 108% and 120%. The RUL predictions estimate a tool life of 2.1 minutes at the end of tool life (see Figure 20). In general, the predictions are better if the tool life occurs at the

$p_{rms}$  value within the interval considered, which can be updated for subsequent tests. The sample prior curves can be generated using the updated values for every test. This would improve the predictions for subsequent tests. Future work will focus on improving the prior by using experimental result data from tests.

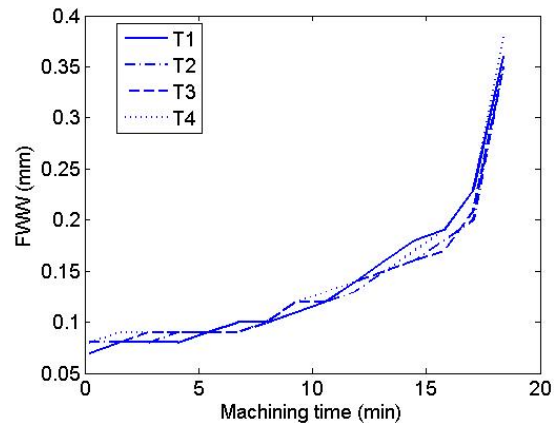


Figure 16. Growth in  $FWW$  with machining time (four teeth).

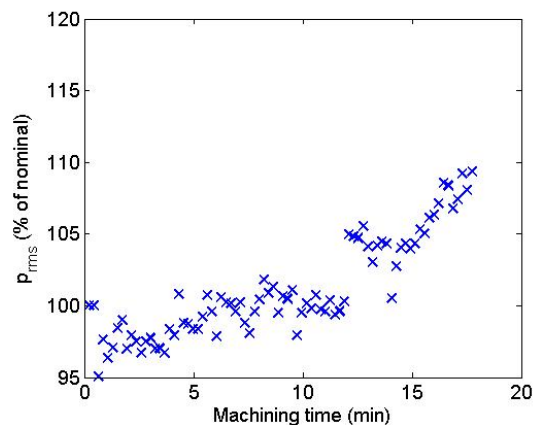


Figure 17.  $p_{rms}$  measured values.

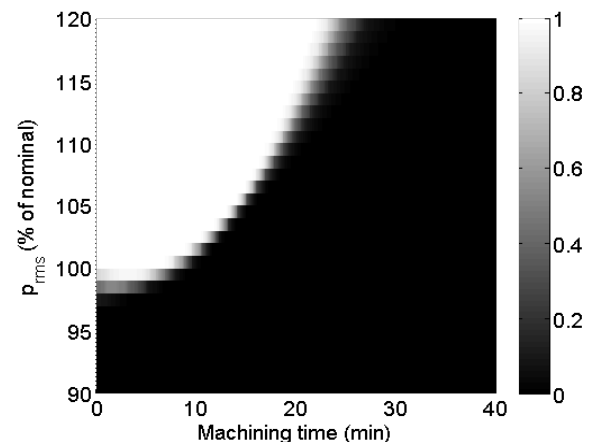


Figure 18. Posterior cdf using  $p_m$  values.

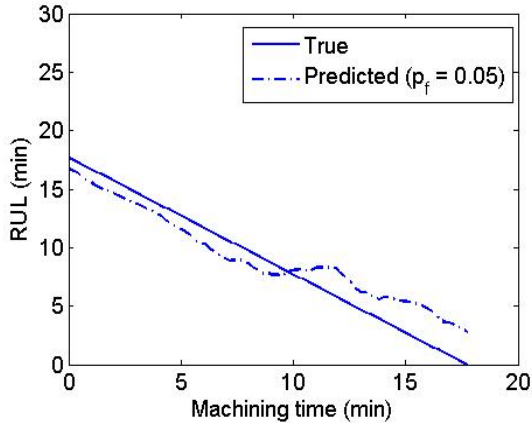


Figure 19. RUL predictions.

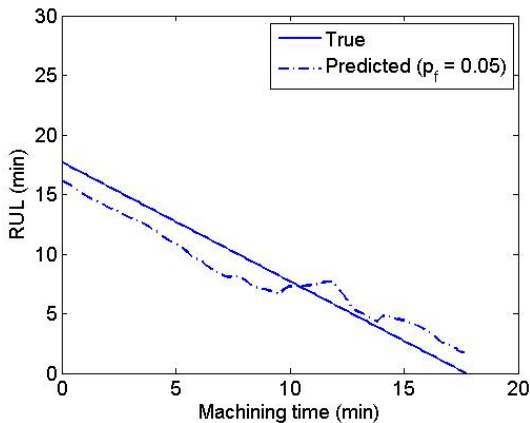


Figure 20. RUL predictions.  $p_{rms}$  value was assumed to be equally likely between 108% and 120% at the end of tool life.

## CONCLUSIONS

A random walk method of Bayesian updating for predicting remaining useful tool life was presented. The root mean square (RMS) value of the spindle power in the time domain was found to be sensitive to tool wear. Random sample power RMS growth curves were generated and the probability of each representing the true curve was updated using measurements. The updated probabilities of the sample curves were used to predict the remaining useful tool life.

The method offers many advantages. First, it incorporates uncertainties in tool life and the normalized power sensor value at the end of tool life. Therefore, the method is robust to outlier points and more reliable than methods that rely on a deterministic threshold sensor value. The method incorporates prior information and does not require a large training data set. The prior information can also be improved after each test. Second, the method is computationally inexpensive and can be incorporated for real-time predictions. To illustrate, using Matlab™ on an Intel i5 processor, the computation takes 1 second. The computation time can be further decreased by

reducing the discrete steps in  $p_{rms}$  values. Third, the remaining useful life percentile (e.g., 95%) can consider user preferences and applications. For example, in an application using expensive parts/tooling, the user can select a conservative RUL percentile (say 99%). On the other hand, for low cost applications, such as roughing, the user can select a low percentile (say 90%). The optimum percentile can be calculated using an expected cost formulation. The risk preferences of the user (risk neutral or risk averse) can also be incorporated. Future work will explore the optimum percentile calculations using decision theory principles. In addition, future work will focus on validation at different machining conditions, speed, feed, axial and radial depths of cut.

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