APPLICATION OF WAVELET ANALYSIS IN HETERODYNE INTERFEROMETRY

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INTRODUCTION
Heterodyne displacement measuring interferometry provides important metrology for applications requiring high resolution and accuracy, such as in the semiconductor manufacturing industry and for linear stage calibration. Ideally, the measurement and reference beams in the interferometer completely separate and recombine, where the phase shift is linear with respect to displacement. In practice, however, periodic error exists due to undesirable frequency mixing. Periodic error is non-cumulative and appears as a function of the target displacement. Typically, both 1st and 2nd order periodic errors occur, which correspond to the number of periods (one or two) per fringe displaced, as shown in FIGURE 1. A displacement fringe corresponds to the wavelength divided by the interferometer fold factor which is determined by the interferometer setup. Ultimately, this error can limit the accuracy to approximately the nanometer level.

Many studies have investigated the measurement and compensation of periodic error, including frequency domain and time domain approaches. For frequency domain approaches, the periodic error is measured by calculating the Fourier transform of the time domain data collected during constant velocity target displacement [1]. This method is not well suited to non-constant velocity profiles. An alternate digital algorithm which can be applied in real-time for constant or non-constant velocity motions is also available for measuring and compensating 1st order periodic error [2].

In this work, an analytical wavelet-based technique is used to analyze periodic error for both constant and non-constant velocity motions and also situations where the periodic error amplitude may not be constant, such as interferometers with fiber delivery.

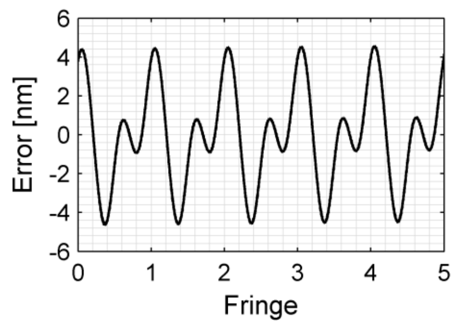

FIGURE 1. Example of 1st and 2nd order periodic error as a function of fringes. Typically, 1st order error has a larger magnitude than 2nd order error.

METHOD
The wavelet transform can be used to analyze time series data that contains non-stationary (variable period) power at multiple frequencies [3]. Wavelet functions refer to either orthogonal or non-orthogonal wavelets. The choice of the appropriate wavelet transform (continuous or discrete) and wavelet function is based on whether the purpose of data analysis is detection or compression [4].

A wavelet function $\psi(\eta)$ is a finite energy function [5] with an average of zero or

$$\int_{-\infty}^{+\infty} \psi(x)dx = 0 \qquad (1)$$

A wavelet family is generated by dilating the mother wavelet via the scale $s > 0$ and translating it via the location $\xi \epsilon R$. This series of wavelets can be expressed as:

$$\psi_{s,\xi}(x) = \frac{1}{\sqrt{s}}\psi\left(\frac{x-\xi}{s}\right) \quad (2)$$

In this research, the continuous wavelet transform (CWT) is used to analyze displacement signals. For a one-dimensional signal $f(x)$, the CWT is

$$W_f(s,\xi) = \int_{-\infty}^{+\infty} f(x)\psi^*_{s,\xi}(x)dx = \int_{-\infty}^{+\infty} f(x)\frac{1}{\sqrt{s}}\psi^*\left(\frac{x-\xi}{s}\right)dx \quad (3)$$

where * indicates the complex conjugate and x represents the time variable.

In practice, EQUATION 3 must be converted from continuous to discrete. For a time series, x_n, with time interval, Δt, where $n = 0 \cdots N-1$, and a wavelet function, $\psi_0(\eta)$, where η is a non-dimensional 'time' parameter, the CWT of a discrete sequence, x_n , is defined as the convolution of this sequence with a scaled and translated version of $\psi_0(\eta)$:

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'}\psi^*[\frac{(n'-n)\Delta t}{s}] \quad (4)$$

where s is the wavelet scale. To demonstrate how the wavelet coefficient amplitudes vary with scales and times, the discrete Fourier transform (DFT) can be applied [5]. The DFT of x_n is

$$\hat{x}_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{-2\pi ikn/N} \quad (5)$$

where N is the number of points in the time series [6] and $k = 0 \cdots N-1$ is the frequency index. Also, the function $\psi(t/s)$ becomes $\hat{\psi}(s\omega)$ in the frequency domain. The wavelet transform is the inverse Fourier transform of the product:

$$W_n(s) = \sum_{n'=0}^{N-1} \hat{x}_k\psi^*(s\omega_k)e^{i\omega_k n\Delta t} \quad (6)$$

where ω_k is the angular frequency.

It is complicated to reconstruct the original time series for the CWT, due to the redundancy in time and scale. This redundancy, however, makes it possible to proceed with a completely different wavelet function such as the delta function [4]. By this approach, deconvolution is used to reconstruct the original time series. Comparison is made between the original and reconstructed signal. In this way, a connection between the wavelet function and the output signal is established.

ANALYSIS

Simulated and experimental displacement signals with periodic error were used to assess the validity of the wavelet-based technique.

Periodic Error Reconstruction

The specific wavelet function adopted in this analysis was the Morlet wavelet. The errors were isolated from the displacement signal by subtracting a least-squares best fit polynomial. In FIGURE 2, the simulation results show that the difference between the original and reconstructed periodic error is small, which demonstrates that the signal can be accurately constructed using wavelet analysis. FIGURE 3 shows measurements with periodic (some low frequency drift remained after the least-squares best fit subtraction). The difference during the reconstruction process is still acceptable.

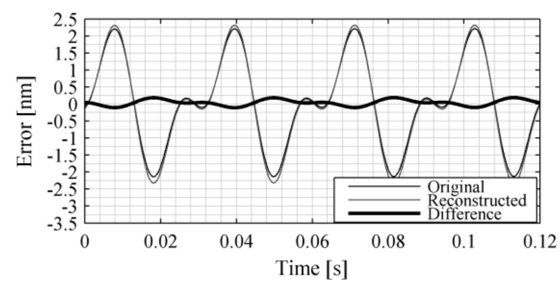

FIGURE 2. The simulated original and reconstructed errors with the difference.

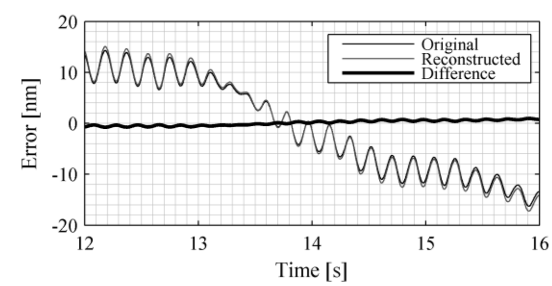

FIGURE 3. The experimental original and reconstructed errors with the difference.

Periodic Error Detection

Here the Haar wavelet was used to detect the periodic errors in the displacement signal. The Haar wavelet is the simplest wavelet and it is not continuous. This property, however, can be an advantage to detect the sudden transitions and sharp rising and falling edges in signals, which is possible in periodic error detection. The signal in FIGURE 4 shows a simulated linear motion

($1.0 \times 10^{-6} m/s$ velocity) with 1st and 2nd order periodic errors.

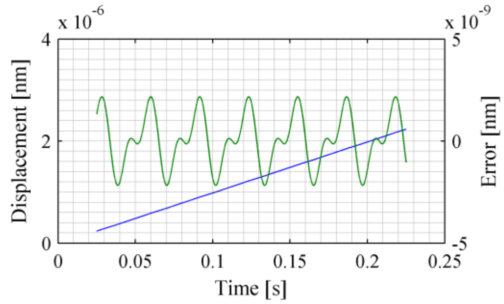

FIGURE 4. The simulated displacement signal with constant velocity and the periodic errors within it.

FIGUREs 5 and 6 give the CWT result with the displacement signal when the peak amplitude of the periodic errors varies.

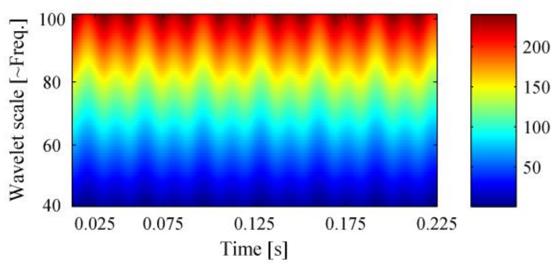

FIGURE 5. The peak amplitudes of 1st and 2nd order periodic errors are $1.28nm$ and $0.95nm$ respectively.

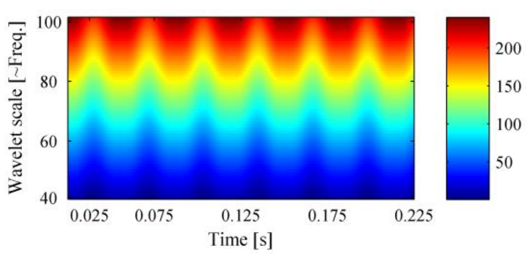

FIGURE 6. The peak amplitudes of 1st and 2nd order periodic errors are $2.99nm$ and $0.37nm$ respectively.

The time intervals between every two closed peaks of the coefficients at the same scale can be used to determine the frequency of the periodic errors. Using the method, the frequency of the periodic error is calculated to be $32Hz$, which matches the simulated frequency value. The velocity of a measured artifact is linearly proportional to the frequency of the periodic errors. Therefore, through wavelet analysis results the periodic error frequency information as well as the velocity of the original motion can be obtained.

The CWT was applied to a simulated accelerated motion of $2.0 \times 10^{-5} m/s^2$ with periodic errors (FIGURE 7). The result is shown in FIGURE 8. Similar to method described previously, the periodic error frequency is observed to increase from approximately $34Hz$ to $45Hz$ during increasing velocity motion.

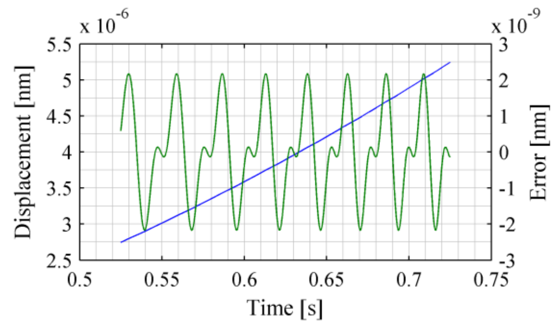

FIGURE 7. The simulated displacement signal with non-constant velocity and the periodic errors within it.

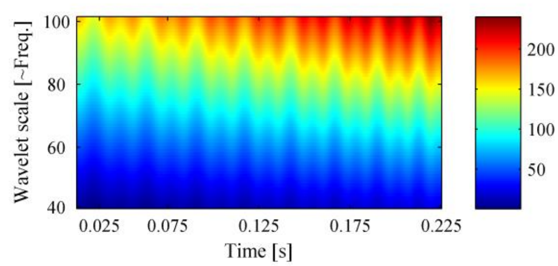

FIGURE 8. The simulated displacement signal with non-constant velocity and the periodic errors within it.

In addition, wavelet analysis was used with experimental data to detect the periodic errors for a heterodyne setup with a sampling rate of $3.2\mu s$. FIGURE 9 shows the displacement and errors.

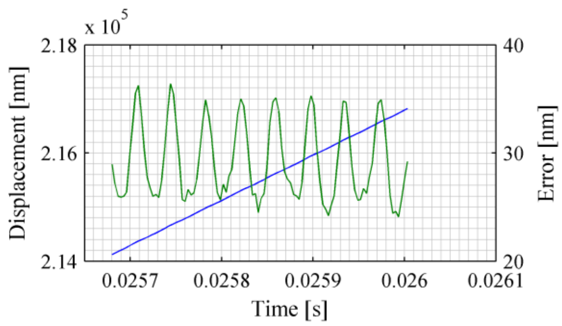

FIGURE 9. The experimental displacement signal with errors.

FIGURE 10 shows the CWT coefficients for the signal depicted in FIGURE 9. FIGURE 11 reveals specific coefficients at scale 10.

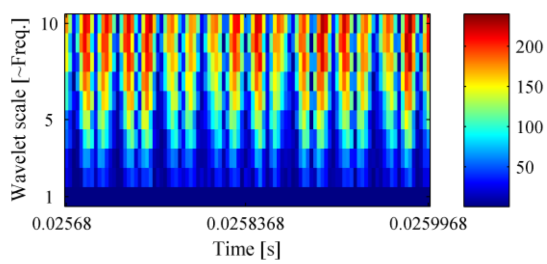

FIGURE 10. The CWT result with the experimental signal.

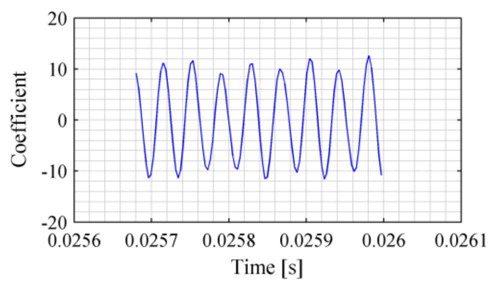

FIGURE 11. The CWT coefficients at scale 10.

Similarly, measuring the average of the time intervals by every two closed peaks, the frequency of the periodic errors obtained here is approximately 26042Hz.

CONCLUSIONS

Wavelet analysis is used in this research to establish an accurate connection between the selected wavelet function and periodic error frequencies/amplitudes in heterodyne interferometer displacement signals. Future work will focus on applying the wavelet analysis in real time and developing an appropriate correction algorithm based on the wavelet coefficients to compensate for periodic errors for both constant and non-constant velocity motion.

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